# Low-Complexity Two-Way AF Relay Design for Millimeter Wave Communication Systems

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***Abstract*—In this paper, we consider the problem of hybrid multi-input multi-output (MIMO) transceiver design for a non-regenerative hybrid two-way amplify-forward (AF) relay based millimeter wave (mmWave) communication system. We propose two low complexity mmWave system designs based on the quality of available channel state information (CSI). We first propose the design of transceivers and a two-way AF relay, by minimizing sum-mean square error (SMSE) under the constraint of total relay transmit power while assuming the perfect channel knowledge. We later extend it to a robust design under the assumption that the transceivers posses only imperfect information about the channel state, where the CSI error is assumed to follow the Gaussian distribution. In both the designs, reduced hardware complexity (introduced due to large number of antenna elements in mmWave system) is achieved by analog-digital hybrid processing by using orthogonal matching pursuit (OMP)-based sparse signal processing. We present numerical results on the performance of both the designs over various dictionaries. The comparison results show that robust design is resilient to the presence of CSI errors. Furthermore, we also demonstrate the convergence of both the proposed algorithms to a limit even though global convergence is hard to prove due to non convex nature of overall optimization problem.**

## I. Introduction

Millimeter wave (mmWave) communication, envisioned as one of the promising technologies for future wireless networks, has the potential to offer the benefit of massive bandwidth for next generation mobile communication networks. With the incredible increase in wireless data traffic, the currently available spectrum at microwave frequencies (upto 6 GHz) would be totally exhausted in near future. Hence the idea of exploiting the under-utilized millimeter wave spectrum (30-300 GHz) for cellular applications is gaining tremendous interest among researchers [1]. At mmWave frequencies, as large number of antennas can be packed in small volume due to small size of elements, it is possible to achieve high beamforming gain and high signal-to-noise ratio (SNR) that overcomes significant pathloss, making MIMO a key parameter for these systems. However, due to the high cost and power consumption of RF chains consisting of amplifiers, mixers, analog-to-digital converters (ADCs) / digital-to-analog converters (DACs), highly compact mmWave systems demand reduced hardware complexity. Hence, unlike the traditional MIMO, where each antenna element is associated with a dedicated RF chain, it is important to investigate methods to reduce the number of RF chains associated with large antenna array in mmWave communication systems. Hybrid MIMO architecture overcomes this limitation by processing the data in two sequential phases, i.e., digital baseband processing followed by analog RF beamforming. This architecture can achieve reduction in complexity by using less number of RF chains. Various techniques to implement hybrid beamforming have been proposed [2] [3]. Phase shifters forming the phased array beamformers are used in order to realise the RF beamformers [4]. In [5], hybrid design is achieved by formulating the model as a sparse approximation mathematical problem which is solved using iterative greedy matching pursuit algorithms. Recently, hybrid design by alternating minimization and least square minimization for mmWave system has been discussed in [6].

mmWaves suffer severe losses such as rain fading, free space path loss, less significant scattering and pronounced coverage and blockage holes due to weaker non line-of-sight paths [7]. The resulting performance degradation can be mitigated to some extent by the use of cooperative relaying techniques. In the literature, relays have been widely studied for conventional MIMO communication systems. Joint precoding/receive filter design in multi-user two-way MIMO relay systems is discussed in [8]. In [9], MMSE based MIMO half-duplex (HD) non-regenerative relay precoder design under channel imperfections has been discussed. Full-duplex (FD) relay based system design with self interference cancellation technique for MIMO system have two users has been discussed [10] and [11]. Recently, studies on application of AF relay in mmWave system has also been reported in [12].

In this paper, we propose non-robust and robust hybrid two-way AF relay-assisted mmWave communication system design considering two users and all units working in half duplex (HD) mode. We focus on joint designs for precoder, relay beamformer and receive filter. We consider two users that can communicate with each other only via relay. We obtain optimal filters by minimizing SMSE for both users under a constraint on total transmit power at the relay. The resulting optimization problems are non-convex in nature. We show that the proposed algorithm converges to a limit and obtain the near optimal filters even though the global convergence is not guaranteed. We later decompose this optimal design to a low complexity analog-digital hybrid design by using OMP-based sparse approximation method. OMP is a well known signal processing method and has been extensively studied in literature [13]. In OMP algorithm, RF beamforming vectors are

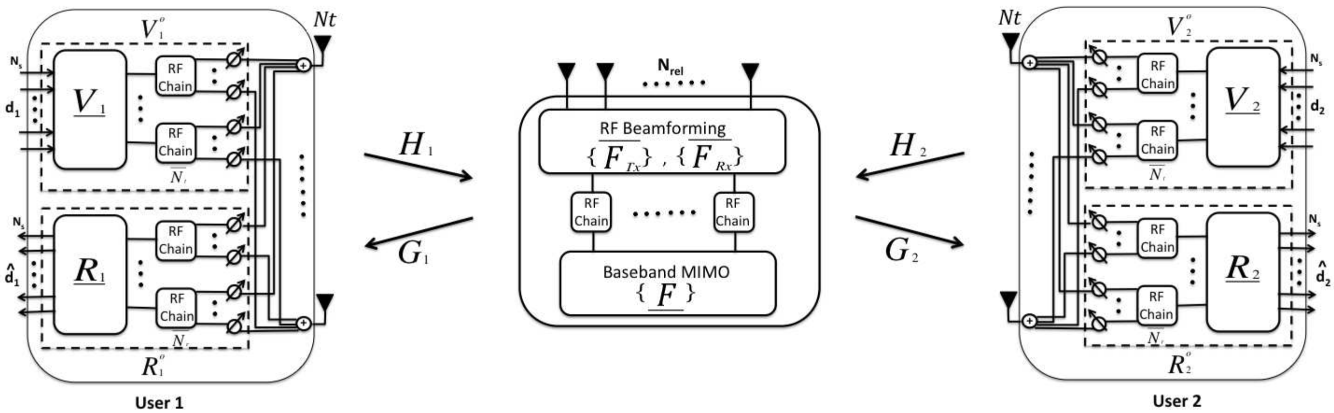


Fig. 1. Hybrid two-way AF relay-assisted system design for mmWave communication

selected from the set of candidate vectors called dictionaries using MMSE criterion and corresponding baseband filter is obtained as a least-square solution. We design this proposed system by considering the availability of perfect CSI.

However, the CSI available with the system is not always perfect due to various factors such as estimation, feedback delays, quantization, etc., which can introduce errors. Hence, the transceivers that are designed assuming the availability of perfect CSI are not likely to achieve desired performance in the presence of CSI errors. Effect of imperfect CSI on the performance of MIMO systems is discussed in [14]. Hence, we extend our design to a robust case where we consider the imperfections in the available channel knowledge and develop a system that is resilient to erroneous CSI for mmWave communication. We evaluate the performance of both the proposed designs for various parameters and results are discussed later in this paper. We also study the performance over different dictionaries composed of eigen beamforming, antenna selection, discrete cosine transform (DCT) and discrete fourier transform (DFT). We compare the proposed designs and it is observed that the robust design is resilient in presence of the CSI errors. The rest of the paper is organized as follows. Sec. II describes the system model. The proposed low complexity designs are discussed in Sec. III and Sec. IV presents the simulation results. Finally, the conclusion is given in Sec. V.

*Notations*: Throughout this paper, we use bold-faced lowercase letters to denote column vectors and bold-faced uppercase letters to denote matrices. $\underline{\mathbf{X}}$ and $\overline{\mathbf{X}}$ implies that the variable $\mathbf{X}$ corresponds to the baseband and RF block, respectively. $\mathbf{C}_{k,t}$ denotes the matrix value for $(k)^{th}$ user at timeslot $t$. $\mathrm{tr}(\cdot)$ denotes the trace operator and $\mathbb{E}\{\cdot\}$ is the expectation operator. $||\cdot||_F$ represents the frobenius norm operator. $vec(\mathbf{X})$ determines the vectorization operation for any matrix $\mathbf{X}$ and $\otimes$ denotes the Kronecker product.

## II. System Model

We consider a hybrid mmWave wireless communication system with two users (U1 and U2) and a non-regenerative relay as shown in Figure 1. Both users communicate with each other only via a two-way relay assuming that there is no direct link between them. All the units are equipped with hybrid MIMO processors and operate in half-duplex mode. A hybrid unit processes the data in two sequential phases, *viz.*, analog RF beamforming and digital baseband processing [2]. Each user either transmits or receives $N_s$ independent data symbols through its corresponding $Nt$ antennas. The information exchange between the two users take place in two time slots. In the first slot, both the sources simultaneously transmit their corresponding data to the relay. In the next time slot, relay re-transmits the signal it received in the previous slot after multiplying by a matrix . The matrices $\mathbf{H}_{i,t}$ and $\mathbf{G}_{i,t}$ represent the channel gains from user $i$ to relay (forward channel) and relay to user $i$ (reverse channel) at time $t$ for $i \in \{1,2\}$ respectively. The signal transmitted by the $i$th user is denoted by the $N_s-$ dimensional column vector $\mathbf{d}_i$ , $i = \{1,2\}$. The relay is assumed to have total of $N_{rel}$ number of antennas from which $Nr_{Tx}$ are used as transmit antennas and $Nr_{Rx}$ as the receive antennas. The number of RF chains associated with each user and the relay unit are $N_{RF}$ and $Nr_{RF}$, respectively, such that $N_s \leq N_{RF} \leq Nt$ and $Nr_{RF} \leq N_{rel}$. Let $\mathbf{V}_i^o, \mathbf{R}_i^o$ $i \in \{1,2\}$, and $\mathbf{F^o}$ be the optimal precoder, receive filter matrix and relay gain matrix in the conventional fully digital MIMO system at the $i^{th}$ user terminal. Then, the hybrid, $N_{RF} \times N_s$ matrix $\underline{\mathbf{V}}_i$ denotes the digital baseband precoder and the $Nt \times N_{RF}$ matrix $\overline{\mathbf{V}}_i$ denotes the RF beamformer at the $i$th transmitter, i$\in$ $\{1,2\}$. The relay receives transmitted signal in first slot through a $Nr_{Rx} \times Nr_{RF}$ RF beamformer $\overline{\mathbf{F}}_{Rx}$ which is then multiplied by $(Nr_{RF} \times Nr_{RF})$ AF relay gain matrix $\underline{\mathbf{F}}$. The resultant signal is forwarded to their respective destination through $Nr_{RF} \times Nr_{Tx}$ RF beamformer $F_{Tx}$ through the relay. Received signal at the destination is processed through an RF beamformer denoted by $N_{RF} \times Nt$ matrix $\overline{\mathbf{R}}_i$ followed by a baseband combiner denoted by the $N_s \times N_{RF}$ matrix $\underline{\mathbf{R}}_i$. We design the hybrid filters to satisfy the following: $\mathbf{V}_i^o = \underline{\mathbf{V}}_i \overline{\mathbf{V}}_i, \mathbf{F}^o = \overline{\mathbf{F}}_{Tx}\underline{\mathbf{F}}\overline{\mathbf{F}}_{Rx}$, and $\mathbf{R}_i^o = \underline{\mathbf{R}}_i^H \overline{\mathbf{R}}_i^H$. The signal received by the relay in time slot t-1 is given by,

$$\mathbf{y}_{r,t-1} = \mathbf{H}_{1,t-1}\mathbf{V}_{1,t-1}\mathbf{x}_{1,t-1} + \mathbf{H}_{2,t-1}\mathbf{V}_{2,t-1}\mathbf{x}_{2,t-1} + \mathbf{n}_r, \quad (1)$$

where, $\mathbf{n}_r$ is additive white Gaussian noise with zero mean and variance $\sigma_n^2\mathbf{I}$ at the receiver, $\mathbf{n}_r \in \mathbb{C}^{Nt \text{ x } 1}$.

TABLE I

| Iterative algorithm computing $\mathbf{F}^0, \mathbf{V}^o,$ *and* $\mathbf{R}^o$ for MSE optimization |
|---|
| 1. Initialize $\mathbf{F}_t, \mathbf{V}_{k,t-1}$ and $\mathbf{R}_{k,t}$ $\quad \forall k \in \{1,2\}$ |
| 2. Repeat, |
| 3. Update $\tilde{\mathbf{F}}_t$ by using $\mathbf{R}_{k,t}, \mathbf{V}_{k,t-1}$ $\quad \forall k \in \{1,2\}$ |
| 4. Update $\alpha_t$ by using $\tilde{\mathbf{F}}_t$ |
| 5. Update $\mathbf{F}_t$ by using $\tilde{\mathbf{F}}_t$ and $\alpha_t$ |
| 6. Update $\mathbf{R}_{k,t}$ by using $\mathbf{F}_t$ and $\mathbf{V}_{k,t-1}$ $\quad \forall k \in \{1,2\}$ |
| 7. Update $\mathbf{V}_{k,t-1}$ by using $\mathbf{F}_t$ and $\mathbf{R}_{k,t}$ $\quad \forall k \in \{1,2\}$ |
| 5. Repeat until convergence. |

In next time slot, the received signal by relay is multiplied by relay gain matrix and transmitted to their respective destinations. In time slot $t$, the transmitted signal by the relay can be given by,

$$\begin{aligned}\mathbf{x}_{r,t} &= \mathbf{F}_t \mathbf{y}_{r,t-1}, \\ &= \mathbf{F}_t \mathbf{H}_{1,t-1} \mathbf{V}_{1,t-1} \mathbf{x}_{1,t-1} + \mathbf{F}_t \mathbf{H}_{2,t-1} \mathbf{V}_{2,t-1} \mathbf{x}_{2,t-1} + \mathbf{F}_t \mathbf{n}_r.\end{aligned} \tag{2}$$

After receiving the signal transmitted by the relay, each user can cancel out its own transmitted signal which appears as self interference. The resultant received signal can be given as follows. for user-1,

$$\mathbf{y}_{1,t} = \mathbf{G}_{1,t} \mathbf{F}_t \mathbf{H}_{2,t-1} \mathbf{V}_{2,t-1} \mathbf{x}_{2,t} + \mathbf{G}_{1,t} \mathbf{F}_t \mathbf{n}_r + \mathbf{n}_1, \tag{3}$$

and for user-2,

$$\mathbf{y}_{2,t} = \mathbf{G}_{2,t} \mathbf{F}_t \mathbf{H}_{1,t-1} \mathbf{V}_{1,t-1} \mathbf{x}_{1,t} + \mathbf{G}_{2,t} \mathbf{F}_t \mathbf{n}_r + \mathbf{n}_2. \tag{4}$$

Hence for any user-k, its received signal can be given by,

$$\mathbf{y}_{k,t} = \mathbf{G}_{k,t} \mathbf{F}_t \mathbf{H}_{\overline{k},t-1} \mathbf{V}_{\overline{k},t-1} \mathbf{x}_{\overline{k},t} + \mathbf{G}_{k,t} \mathbf{F}_t \mathbf{n}_r + \mathbf{n}_k. \tag{5}$$

where, $\overline{k}$ denotes other user i.e. when $k = 1$, $\overline{k} = 2$ and vice verse. We design the above system assuming that the transceiver possesses perfect channel state information. We later extend the design to robust design by assuming imperfections in the channel state information. Specifically, the CSI in this case can be modeled as,

$$\mathbf{C} = \widehat{\mathbf{C}} + \mathbf{\Delta}, \tag{6}$$

where $\widehat{\mathbf{C}}$ is the estimated CSI available with the transceivers, and $\mathbf{\Delta} \sim \mathcal{N}(0, \sigma_E^2 \mathbf{I})$ denotes the corresponding error in the CSI.

## III. Low-Complexity AF Two-Way Relay Design

In this section, we present the design of a two-way relay-assisted hybrid mmWave communication system with 2 users in non-line of sight path, communicating only through a relay. We first design the fully-digital optimal precoder $\mathbf{V}_i^o$, optimal receive filter $\mathbf{R}_i^o$ and optimal relay filter $\mathbf{F}^o$ for $i \in \{1,2\}$ by minimizing sum-MSE with a constraint on total relay transmission power. Subsequently, from these fully-digital optimal filters, we obtain the hybrid precoder matrices $\overline{\mathbf{V}}_k$ and $\underline{\mathbf{V}}_k$, hybrid receive filter matrices $\overline{\mathbf{R}}_k$, and $\underline{\mathbf{R}}_k$ and hybrid relay filter matrices $\overline{\mathbf{F}}_{Tx}, \overline{\mathbf{F}}_{Rx},$ and $\underline{\mathbf{F}}$ with lower complexities using OMP-based sparse approximation technique.

### A. Conventional Joint Transceivers and Relay Filter Design

We design the optimal system by minimizing the overall sum-MSE under the constraint on total relay transmit power. Thus, the optimization problem can be expressed as:

$$\min_{\mathbf{F}, \mathbf{V}_i, \mathbf{R}_i} \text{SMSE} \quad \text{s.t: } ||\mathbf{x}_{r,t}||_F^2 \le (N r_{Tx} P_r), \tag{7}$$

where, $P_r$ is the total transmit power at each antenna of the relay. We consider an extra variable $\alpha$ for derivational simplicity of the problem. For the proposed system, sum-MSE at timeslot $t$ can be formulated as:

$$\begin{aligned}\text{SMSE}_t = \mathbb{E}\big[||\mathbf{x}_{2,t-1} - \alpha^{-1} \mathbf{R}_{1,t} \mathbf{y}_{1,t}||^2\big] \\ + \mathbb{E}\big[||\mathbf{x}_{1,t-1} - \alpha^{-1} \mathbf{R}_{2,t} \mathbf{y}_{2,t}||^2\big],\end{aligned} \tag{8}$$

$$\begin{aligned} = \operatorname{tr}\Big(&\mathbf{I} - \alpha^{-1} \mathbf{R}_{1,t} \mathbf{G}_{1,t} \mathbf{F}_t \mathbf{H}_{2,t-1} \mathbf{V}_{2,t-1} \\ &-\alpha^{-1} \mathbf{V}_{2,t-1}^H \mathbf{H}_{2,t-1}^H \mathbf{F}_t^H \mathbf{G}_{1,t}^H \mathbf{R}_{1,t}^H \\ &+\alpha^{-2} \mathbf{R}_{1,t} \mathbf{G}_{1,t} \mathbf{F}_t \mathbf{H}_{2,t-1} \mathbf{V}_{2,t-1} \mathbf{V}_{2,t-1}^H \mathbf{H}_{2,t-1}^H \mathbf{F}_t^H \mathbf{G}_{1,t}^H \mathbf{R}_{1,t}^H \\ &+\alpha^{-2} \sigma_r^2 \mathbf{R}_{1,t} \mathbf{G}_{1,t} \mathbf{F}_t \mathbf{F}_t^H \mathbf{G}_{1,t}^H \mathbf{R}_{1,t}^H + \alpha^{-2} \sigma_1^2 \mathbf{R}_{1,t} \mathbf{R}_{1,t}^H \\ &+\mathbf{I} - \alpha^{-1} \mathbf{R}_{2,t} \mathbf{G}_{2,t} \mathbf{F}_t \mathbf{H}_{1,t-1} \mathbf{V}_{1,t-1} \\ &-\alpha^{-1} \mathbf{V}_{1,t-1}^H \mathbf{H}_{1,t-1}^H \mathbf{F}_t^H \mathbf{G}_{2,t}^H \mathbf{R}_{2,t}^H \\ &+\alpha^{-2} \mathbf{R}_{2,t} \mathbf{G}_{2,t} \mathbf{F}_t \mathbf{H}_{1,t-1} \mathbf{V}_{1,t-1} \mathbf{V}_{1,t-1}^H \mathbf{H}_{1,t-1}^H \mathbf{F}_t^H \mathbf{G}_{2,t}^H \mathbf{R}_{2,t}^H \\ &+\alpha^{-2} \sigma_r^2 \mathbf{R}_{2,t} \mathbf{G}_{2,t} \mathbf{F}_t \mathbf{F}_t^H \mathbf{G}_{2,t}^H \mathbf{R}_{2,t}^H + \alpha^{-2} \sigma_2^2 \mathbf{R}_{2,t} \mathbf{R}_{2,t}^H \Big).\end{aligned} \tag{9}$$

Optimization problem given in (7) after substitution (9) can be solved by using Karush-Kuhn-Tucker conditions. The Langrangian function $J(\alpha, \mathbf{V}_{1,t-1}, \mathbf{V}_{2,t-1}, \mathbf{R}_{1,t}, \mathbf{R}_{2,t}, \mathbf{F}_t)$ associated with the above optimization problem is given by,

$$\begin{aligned} J = \Big(&\mathbf{I} - \mathbf{R}_{1,t} \mathbf{G}_{1,t} \tilde{\mathbf{F}}_t \mathbf{H}_{2,t-1} \mathbf{V}_{2,t-1} - \mathbf{V}_{2,t-1}^H \mathbf{H}_{2,t-1}^H \tilde{\mathbf{F}}_t^H \\ &\mathbf{G}_{1,t}^H \mathbf{R}_{1,t}^H + \mathbf{R}_{1,t} \mathbf{G}_{1,t} \tilde{\mathbf{F}}_t \mathbf{H}_{2,t-1} \mathbf{V}_{2,t-1} \mathbf{V}_{2,t-1}^H \mathbf{H}_{2,t-1}^H \tilde{\mathbf{F}}_t^H \\ &\mathbf{G}_{1,t}^H \mathbf{R}_{1,t}^H + \sigma_r^2 \mathbf{R}_{1,t} \mathbf{G}_{1,t} \tilde{\mathbf{F}}_t \tilde{\mathbf{F}}_t^H \mathbf{G}_{1,t}^H \mathbf{R}_{1,t}^H + \alpha^{-2} \sigma_1^2 \mathbf{R}_{1,t} \mathbf{R}_{1,t}^H \\ &+\mathbf{I} - \mathbf{R}_{2,t} \mathbf{G}_{2,t} \tilde{\mathbf{F}}_t \mathbf{H}_{1,t-1} \mathbf{V}_{1,t-1} - \mathbf{V}_{1,t-1}^H \mathbf{H}_{1,t-1}^H \tilde{\mathbf{F}}_t^H \mathbf{G}_{2,t}^H \\ &\mathbf{R}_{2,t}^H + \mathbf{R}_{2,t} \mathbf{G}_{2,t} \tilde{\mathbf{F}}_t \mathbf{H}_{1,t-1} \mathbf{V}_{1,t-1} \mathbf{V}_{1,t-1}^H \mathbf{H}_{1,t-1}^H \tilde{\mathbf{F}}_t^H \mathbf{G}_{2,t}^H \\ &\mathbf{R}_{2,t}^H + \sigma_r^2 \mathbf{R}_{2,t} \mathbf{G}_{2,t} \tilde{\mathbf{F}}_t \tilde{\mathbf{F}}_t^H \mathbf{G}_{2,t}^H \mathbf{R}_{2,t}^H + \alpha^{-2} \sigma_2^2 \mathbf{R}_{2,t} \mathbf{R}_{2,t}^H \\ &+\lambda\big(\alpha^2 \operatorname{tr}\big(\tilde{\mathbf{F}}_t \mathbf{H}_{1,t-1} \mathbf{V}_{1,t-1} \mathbf{V}_{1,t-1}^H \mathbf{H}_{1,t-1}^H \tilde{\mathbf{F}}_t^H \\ &+\tilde{\mathbf{F}}_t \mathbf{H}_{2,t-1} \mathbf{V}_{2,t-1} \mathbf{V}_{2,t-1}^H \mathbf{H}_{2,t-1}^H \tilde{\mathbf{F}}_t^H + \sigma_r^2 \tilde{\mathbf{F}}_t \tilde{\mathbf{F}}_t^H\big) \\ &+ N r_{Tx} P_r\big)\Big),\end{aligned} \tag{10}$$

where, relay beamforming matrix is split as $\mathbf{F} = \alpha \tilde{\mathbf{F}}$, for simplicity. Formulated problem is not convex in optimization variables, thus global optimized solution cannot be achieved. Coordinate descent method, wherein each variable is minimized considering other variables to be fixed, is used to solve this problem. An iterative algorithm given in Table I is used to jointly obtain the sub-optimal solution of the proposed problem. Consider the minimization with respect to receive filter $\mathbf{R}_i$ while keeping other variables fixed, so for user 1, differentiating the lagrangian w.r.t $\mathbf{R}_1^H$ and equating it to zero, we get,

$$\begin{aligned}\frac{\partial J}{\partial \mathbf{R}_{1,t}^H} &= -\mathbf{V}_{2,t-1}^H \mathbf{H}_{2,t-1}^H \tilde{\mathbf{F}}_t^H \mathbf{G}_{1,t}^H + \mathbf{R}_{1,t} \mathbf{G}_{1,t} \tilde{\mathbf{F}}_t \mathbf{H}_{2,t-1} \\ &\mathbf{V}_{2,t-1} \mathbf{V}_{2,t-1}^H \mathbf{H}_{2,t-1}^H \tilde{\mathbf{F}}_t^H \mathbf{G}_{1,t}^H + \sigma_r^2 \mathbf{R}_{1,t} \mathbf{G}_{1,t} \tilde{F}_t \tilde{F}_t^H \mathbf{G}_{1,t}^H \\ &+ \alpha^2 \sigma_r^2 \mathbf{R}_{1,t} = 0.\end{aligned}$$

TABLE II

OMP-based iterative algorithm for hybrid precoder and receive filter design

Require $\mathbf{P}_k^o, \mathbf{\Phi}, \mathbf{S}_{BF}$

1: $\overline{\mathbf{Q}}_k = [\,]$
2: $\mathbf{A}_0 = \mathbf{P}_k^o$
3: for $i = 1$ to $N_{RF}$ do
4: $\quad \mathbf{\Psi}_{i-1} = (\mathbf{\Phi}\mathbf{S}_{BF})^H(\mathbf{\Phi}\mathbf{A}_{i-1})$
5: $\quad l = arg\ max_{m=1...M}(\mathbf{\Psi}_{i-1}\mathbf{\Psi}_{i-1}^H)_{m,m}$
6: $\quad \underline{\mathbf{Q}}_k = [\overline{\mathbf{Q}}_k|\mathbf{\Phi}(:,k)]$
7: $\quad \underline{\mathbf{Q}}_k = (\overline{\mathbf{Q}}_k^H\overline{\mathbf{Q}}_k)^{-1}\overline{\mathbf{Q}}_k^H\mathbf{P}_k^o$
8: $\quad \mathbf{A}_i = \frac{\mathbf{P}_k^o - \overline{\mathbf{Q}}_k\underline{\mathbf{Q}}_k}{\|\mathbf{P}_k^o - \overline{\mathbf{Q}}_k\underline{\mathbf{Q}}_k\|_F}$
9: end for
10: $\underline{\mathbf{Q}} = \sqrt{N_s}\frac{\mathbf{Q}}{\|\overline{\mathbf{Q}}\underline{\mathbf{Q}}H\|_F}$
11: return $\overline{\mathbf{Q}}, \underline{\mathbf{Q}}$

Precoder: $\overline{N} = \overline{N}_t, \mathbf{P}^o = \mathbf{V}^o, \mathbf{\Phi} = \mathbf{\Gamma}_{\mathbf{y}_k^r}^{\frac{1}{2}}$,
$\overline{\mathbf{Q}} = \overline{\mathbf{V}}$, and $\underline{\mathbf{Q}} = \underline{\mathbf{V}}$

Receive filter: $\overline{N} = \overline{N}_r, \mathbf{P}^o = \mathbf{R}^{oH}, \mathbf{\Phi} = \mathbf{\Gamma}_{\mathbf{y}_k}^{\frac{1}{2}}$ ,
$\overline{\mathbf{Q}} = \overline{\mathbf{R}}$, and $\underline{\mathbf{Q}} = \underline{\mathbf{R}}$

Thus,

$$\mathbf{R}_{1,t} = \mathbf{V}_{2,t-1}^H\mathbf{H}_{2,t-1}^H\tilde{\mathbf{F}}_t^H\mathbf{G}_{1,t}^H\big(\mathbf{G}_{1,t}\tilde{\mathbf{F}}_t\mathbf{H}_{2,t-1}\mathbf{V}_{2,t-1}\mathbf{V}_{2,t-1}^H \mathbf{H}_{2,t-1}^H\tilde{\mathbf{F}}_t^H\mathbf{G}_{1,t}^H + \sigma_r^2\mathbf{G}_{1,t}\tilde{\mathbf{F}}_t\tilde{\mathbf{F}}_t^H.\mathbf{G}_{1,t}^H + \alpha^2\sigma_r^2\mathbf{I}\big)^{-1}. \tag{11}$$

Similarly, for user 2,

$$\mathbf{R}_{2,t} = \mathbf{V}_{1,t-1}^H\mathbf{H}_{1,t-1}^H\tilde{\mathbf{F}}_t^H\mathbf{G}_{2,t}^H(\mathbf{G}_{2,t}\tilde{\mathbf{F}}_t\mathbf{H}_{1,t-1}\mathbf{V}_{1,t-1}\mathbf{V}_{1,t-1}^H \mathbf{H}_{1,t-1}^H\tilde{\mathbf{F}}_t^H\mathbf{G}_{2,t}^H + \sigma_r^2\mathbf{G}_{2,t}\tilde{\mathbf{F}}_t\tilde{\mathbf{F}}_t^H\mathbf{G}_{2,t}^H + \alpha^2\sigma_r^2\mathbf{I})^{-1}. \tag{12}$$

Next, consider the minimization w.r.t. precoder matrix. Thus, for user 1,

$$\frac{\partial J}{\partial \mathbf{V}_{1,t-1}^H} = -\mathbf{H}_{1,t-1}^H\tilde{\mathbf{F}}_t^H\mathbf{G}_{2,t}^H\mathbf{R}_{2,t}^H + \mathbf{H}_{1,t-1}^H\tilde{\mathbf{F}}_t^H\mathbf{G}_{2,t}^H\mathbf{R}_{2,t}^H\mathbf{R}_{2,t} \mathbf{G}_{2,t}\tilde{\mathbf{F}}_t\mathbf{H}_{1,t-1}\mathbf{V}_{1,t-1} + \lambda\alpha^2\mathbf{H}_{1,t-1}^H\tilde{\mathbf{F}}_t^H\tilde{\mathbf{F}}_t\mathbf{H}_{1,t-1}\mathbf{V}_{1,t-1} = 0.$$

Thus,

$$\mathbf{V}_{1,t-1} = (\mathbf{H}_{1,t-1}^H\tilde{\mathbf{F}}_t^H\mathbf{G}_{2,t}^H\mathbf{R}_{2,t}^H\mathbf{R}_{2,t-1}\mathbf{G}_{2,t}\tilde{\mathbf{F}}_t\mathbf{H}_{1,t-1} + \lambda\alpha^2\mathbf{H}_{1,t-1}^H\tilde{\mathbf{F}}_t^H\tilde{\mathbf{F}}_t\mathbf{H}_{1,t-1})^{-1}\mathbf{H}_{1,t-1}^H\tilde{\mathbf{F}}_t^H\mathbf{G}_{2,t}^H\mathbf{R}_{2,t}^H. \tag{13}$$

Similarly, for user 2,

$$\mathbf{V}_{2,t-1} = (\mathbf{H}_{2,t-1}^H\tilde{\mathbf{F}}_t^H\mathbf{G}_{1,t}^H\mathbf{R}_{1,t}^H\mathbf{R}_{1,t}\mathbf{G}_{1,t}\tilde{\mathbf{F}}_t\mathbf{H}_{2,t-1} + \lambda\alpha^2\mathbf{H}_{2,t-1}^H\tilde{\mathbf{F}}_t^H\tilde{\mathbf{F}}_t\mathbf{H}_{2,t-1})^{-1}\mathbf{H}_{2,t-1}^H\tilde{\mathbf{F}}_t^H\mathbf{G}_{1,t}^H\mathbf{R}_{1,t}^H. \tag{14}$$

Also, differentiating the lagrangian in equation (10) w.r.t $\alpha$ and $\lambda$ and solving the resulting expressions, we obtain the values for both of them as follows:

$$\frac{\partial J}{\partial \alpha} = -2\alpha^3\sigma_1^2\mathrm{tr}\left(\mathbf{R}_{1,t}\mathbf{R}_{1,t}^H\right) - 2\alpha^3\sigma_2^2\mathrm{tr}\left(\mathbf{R}_{2,t}\mathbf{R}_{2,t}^H\right) + 2\alpha\lambda\mathrm{tr}[\tilde{\mathbf{F}}_t\mathbf{H}_{1,t-1}\mathbf{V}_{1,t-1}\mathbf{V}_{1,t-1}^H\mathbf{H}_{1,t-1}^H\tilde{\mathbf{F}}_t^H + \tilde{\mathbf{F}}_t\mathbf{H}_{2,t-1}\mathbf{V}_{2,t-1}\mathbf{V}_{2,t-1}^H\mathbf{H}_{2,t-1}^H\tilde{\mathbf{F}}_t^H + \sigma_r^2\tilde{\mathbf{F}}_t\tilde{\mathbf{F}}_t^H] = 0,$$

which can be re-written as,

$$\alpha^4\lambda\mathrm{A}_3 = \sigma^2 A_1,$$

$$\text{and, } \lambda\alpha^4 = \frac{A_1}{A_3}, \tag{15}$$

where,

$$A_1 = \sigma^2\mathrm{tr}\left(\mathbf{R}_{1,t}\mathbf{R}_{1,t}^H + \mathbf{R}_{2,t}\mathbf{R}_{2,t}^H\right),$$

$$A_3 = \mathrm{tr}[\tilde{\mathbf{F}}_t\mathbf{H}_{1,t-1}\mathbf{V}_{1,t-1}\mathbf{V}_{1,t-1}^H\mathbf{H}_{1,t-1}^H\tilde{\mathbf{F}}_t^H + \tilde{\mathbf{F}}_t\mathbf{H}_{2,t-1} \mathbf{V}_{2,t-1}\mathbf{V}_{2,t-1}^H\mathbf{H}_{2,t-1}^H\tilde{\mathbf{F}}_t^H + \sigma_r^2\tilde{\mathbf{F}}_t\tilde{\mathbf{F}}_t^H].$$

Solving for $\lambda$,

$$\frac{\partial J}{\partial \lambda} = \alpha^2 A_3 - \mathrm{Nr_{Tx}P_r} = 0,$$

$$\implies \alpha^2 = \frac{\mathrm{Nr_{Tx}\ P_r}}{A_3}.$$

From (15), we have,

$$\lambda\alpha^2 = \frac{A_1}{\mathrm{Nr_TxP_r}}. \tag{16}$$

Solving for relay filter,

$$\frac{\partial J}{\partial \tilde{\mathbf{F}}_t^H} = -\mathbf{C} + \mathbf{D1}\tilde{\mathbf{F}}_t\mathbf{D2} + \mathbf{Q1}\tilde{\mathbf{F}}_t + \mathbf{S1}\tilde{\mathbf{F}}_t\mathbf{S2} + \mathbf{P1}\tilde{\mathbf{F}}_t + \mathbf{X1}\tilde{\mathbf{F}}\mathbf{X2} + \mathbf{L1}\tilde{\mathbf{F}}_t\mathbf{L2} + \mathbf{M1}\tilde{\mathbf{F}}_t = 0, \tag{17}$$

where,

$$\mathbf{C} = \mathbf{G}_{1,t}^H\mathbf{R}_{1,t}^H\mathbf{V}_{2,t-1}\mathbf{H}_{2,t-1}^H + \mathbf{G}_{2,t}^H\mathbf{R}_{1,t}^H\mathbf{V}_{1,t-1}^H\mathbf{H}_{1,t-1}^H,$$
$$\mathbf{D1} = \mathbf{G}_{1,t}^H\mathbf{R}_{1,t}^H\mathbf{R}_{1,t}\mathbf{G}_{1,t},$$
$$\mathbf{D2} = \mathbf{H}_{2,t-1}\mathbf{V}_{2,t-1}\mathbf{V}_{2,t-1}^H\mathbf{H}_{2,t-1}^H,$$
$$\mathbf{Q1} = \sigma_r^2\mathbf{G}_{1,t}^H\mathbf{R}_{1,t}^H\mathbf{R}_{1,t}\mathbf{G}_{1,t},$$
$$\mathbf{S1} = \mathbf{G}_{2,t}^H\mathbf{R}_{2,t}^H\mathbf{R}_{2,t},$$
$$\mathbf{S2} = \mathbf{H}_{1,t-1}\mathbf{V}_{1,t-1}\mathbf{V}_{1,t-1}^H\mathbf{H}_{t,t-1}^H,$$
$$\mathbf{P1} = \sigma_r^2\mathbf{G}_{2,t}^H\mathbf{R}_{2,t}^H\mathbf{R}_{2,t}\mathbf{G}_{2,t},$$
$$\mathbf{X1} = \lambda\sigma_r^2\mathbf{I}_{Nr_{Rx}},$$
$$\mathbf{X2} = \mathbf{H}_{1,t-1}\mathbf{V}_{1,t}\mathbf{V}_{1,t}^H\mathbf{H}_{1,t-2}^H,$$
$$\mathbf{L1} = \lambda\sigma_r^2\mathbf{I}_{Nr_{Rx}},$$
$$\mathbf{L2} = \mathbf{H}_{2,t-1}\mathbf{V}_{2,t-1}\mathbf{V}_{2,t-1}^H\mathbf{H}_{2,t-1}^H, \text{ and}$$
$$\mathbf{M1} = \lambda\alpha^2\sigma_r^2\mathbf{I}_{Nr_{Rx}}.$$

Performing the vectorization operation on (17) we get,

$$vec(\mathbf{D1}\tilde{\mathbf{F}}_t\mathbf{D}2 + \mathbf{Q1}\tilde{\mathbf{F}}_t + \mathbf{S1}\tilde{\mathbf{F}}_t\mathbf{S}2 + \mathbf{P1}\tilde{\mathbf{F}}_t + \mathbf{X1}\tilde{\mathbf{F}}\mathbf{X}2 + \mathbf{L}1\tilde{\mathbf{F}}_t\mathbf{L}2 + \mathbf{M1}\tilde{\mathbf{F}}_t)_= vec(\mathbf{C}). \tag{18}$$

For any three matrices, $\mathbf{A}_1$, $\mathbf{A}_2$, and $\mathbf{A}_3$ of suitable dimensions, $\mathrm{vec}(\mathbf{A}_1\mathbf{A}_2\mathbf{A}_3) = \mathbf{A_3}^T \otimes \mathbf{A}_1\mathrm{vec}(\mathbf{A}_2)$. Applying this identity in (18), we get

$$\widehat{\mathbf{f}}_t = \mathbf{Z}^{-1}\mathbf{f}, \tag{19}$$

where $\widehat{\mathbf{f}}_t = vec(\tilde{\mathbf{F}}_t)$ and $\mathbf{f} = \mathbf{vec}(\mathbf{C})$ and

$$\mathbf{Z} = \mathbf{D}2^T \otimes \mathbf{D1} + \mathbf{I}_{Nt_{Tx}}^T \otimes \mathbf{Q1} + \mathbf{S2}^T \otimes \mathbf{S1} + \mathbf{I}_{Nt_{Tx}}^T \otimes \mathbf{P1} + \mathbf{X}2^T \otimes \mathbf{X1} + \mathbf{L}2^T \otimes \mathbf{L1} + \mathbf{I}_{Nr_{Tx}}^T \otimes \mathbf{M1}.$$

We can obtain $\tilde{\mathbf{F}}_t$ from $\widehat{\mathbf{f}}_t$ in (19) and having done that we can compute optimal relay filter as,

$$\mathbf{F}_t = \alpha\tilde{\mathbf{F}}_t. \tag{20}$$

### *B. OMP-Based Low Complexity Hybrid Solution*

In this section, we develop solution for hybrid precoder, receive filter and relay gain filter design proposed communication system. We decompose the previously obtained optimal matrices $\{\mathbf{V}_i^o\}$, $\{\mathbf{R}_i^o\}$ and $\{\mathbf{F}^o\}$ into their corresponding hybrid processors. To achieve this, we adopt the orthogonal matching pursuit (OMP)-based sparse approximation technique. We first formulate the OMP sparse signal processing problem for all these filters, solving which results in the corresponding hybrid matrices. The design of the hybrid system is discussed in subsequent subsections.

*1) Hybrid OMP-Based Precoding/Receive Filter design:* Given the optimal filter in (11), (12), and (13), (14), we obtain the proposed hybrid processors decomposing it to RF/baseband filters by using OMP sparse approximation technique. OMP algorithm has been extensively studied in literature and used for various signal processing applications [2] [5] [13]. OMP is a greedy approach which iteratively computes the RF and baseband matrices. It selects the RF beamforming vectors from dictionary that is most strongly correlated to the residue computed at each iteration. On the other hand, the baseband matrices are computed by solving the least square problem. In general for any user k, considering the respective optimal MMSE receive filter solution given in (11) or (12), the optimal MMSE solution can be rewritten as,

$$\mathbf{R}_k^o = \mathbf{\Gamma}_{\mathbf{y}_k}\mathbf{\Gamma}_{\mathbf{y}_k\mathbf{s}_k}^{-1}, \tag{21}$$

where $\mathbf{\Gamma}_{\mathbf{y}_k} = \mathbb{E}[\mathbf{y}_k\mathbf{y}_k^H]$ and $\mathbf{\Gamma}_{\mathbf{y}_k\mathbf{d}_k} = \mathbb{E}[\mathbf{y}_k\mathbf{d}_k^H]$, and the baseband receive filter matrix can be written as

$$\underline{\mathbf{R}}_k^o = \mathbf{\Gamma}_{\mathbf{z}_k}^{-1}\mathbf{\Gamma}_{\mathbf{z}_k\mathbf{d}_k} = \left(\overline{\mathbf{R}}_k^H\mathbf{\Gamma}_{\mathbf{y}_k}\overline{\mathbf{R}}_k\right)^{-1}\overline{\mathbf{R}}_k^H\mathbf{\Gamma}_{\mathbf{y}_k\mathbf{d}_k}, \tag{22}$$

where $\mathbf{\Gamma}_{\mathbf{z}_k} = \mathbb{E}[\mathbf{z}_k\mathbf{y}_k^H]$ and $\mathbf{\Gamma}_{\mathbf{z}_k\mathbf{d}_k} = \mathbb{E}[\mathbf{z}_k\mathbf{d}_k^H]$. The OMP sparse problem for receiver can be formulated as,

$$\overline{\mathbf{R}}_k^o = \underset{\overline{\mathbf{R}}_k}{\arg\min}\,\mathbb{E}\|\mathbf{d}_k - \underline{\mathbf{R}}_k^{oH}\overline{\mathbf{R}}_k\|^2, \tag{23}$$

which can be rewritten as,

$$\overline{\mathbf{R}}_k^o = \underset{\overline{\mathbf{R}}_k^o}{\arg\min}\,\|\mathbf{\Gamma}_{\mathbf{y}_k}^{\frac{1}{2}}\mathbf{R}_k^o - \mathbf{\Gamma}_{\mathbf{y}_k}^{\frac{1}{2}}\overline{\mathbf{R}}_k\underline{\mathbf{R}}_k^o\|_F^2. \tag{24}$$

Introducing dictionary $\mathbf{S}_{BF}$, the optimization problem can be rephrased as,

$$\tilde{\underline{\mathbf{R}}}_k^o = \underset{\tilde{\underline{\mathbf{R}}}_k}{\arg\min}\,\|\mathbf{\Gamma}_{\mathbf{y}_k}^{\frac{1}{2}}\mathbf{R}_k^o - \mathbf{\Gamma}_{\mathbf{y}_k}^{\frac{1}{2}}\mathbf{S}_{BF}\tilde{\underline{\mathbf{R}}}_k^o\|_F^2, \tag{25}$$

$$\textit{s.t. } \|\text{diag }(\tilde{\underline{\mathbf{R}}}_k^o\tilde{\underline{\mathbf{R}}}_k^{oH})\|_0 = \overline{N}_r.$$

Similarly, the optimization problem for designing sparse precoder matrix can be written as,

$$\overline{\mathbf{V}}_k^o = \underset{\overline{\mathbf{V}}_k}{\arg\min}\,\|\mathbf{\Gamma}_{\mathbf{y}_k^r}^{\frac{1}{2}}\mathbf{V}_k^o - \mathbf{\Gamma}_{y_k^r}^{\frac{1}{2}}\overline{\mathbf{V}}_k\underline{\mathbf{V}}_k^o\|_F^2, \tag{26}$$

and in terms of dictionary

$$\overline{\mathbf{V}}_k^o = \underset{\tilde{\mathbf{V}}_k}{\arg\min}\,\|\mathbf{\Gamma}_{\mathbf{y}_k^r}^{\frac{1}{2}}\mathbf{V}_k^o - \mathbf{\Gamma}_{\mathbf{y}_k^r}^{\frac{1}{2}}\mathbf{S}_{BF}\tilde{\mathbf{V}}_k^o\|_F^2, \tag{27}$$

$$\text{s.t.} \quad \|\text{diag }(\tilde{\underline{\mathbf{V}}}_k^o\tilde{\underline{\mathbf{V}}}_k^{oH})\|_0 = \overline{N}_t \text{ and } \|\tilde{\underline{\mathbf{V}}}_k\|_F^2 = \|\mathbf{V}_k^o\|_F^2.$$

As shown in Table-II we have used OMP-based iterative algorithm at the Tx and Rx to obtain an optimal solution. It is a generalized algorithm for obtaining $\underline{\mathbf{V}}_k, \overline{\mathbf{V}}_k$ and $\underline{\mathbf{R}}_k, \overline{\mathbf{R}}_k$ by passing the appropriate parameters. The input parameter $\mathbf{S}_{BF}$ represents the set of candidate beamforming vectors acquired from different dictionary sets.

TABLE III

| OMP based Hybrid Relay for MIMO |
|---|
| Require $\mathbf{F}^o, \mathbf{S}_{Rx-BF}, \mathbf{S}_{Tx-BF}$, |
| $\mathbf{F}_{Rx-RF} = \mathbf{F}_{Tx-RF} = []$ |
| $\mathbf{F}_{res} = \mathbf{F}^o$ |
| for $i = 1$ to $Nr_{RF}$ do |
| $\quad \mathbf{\Psi}_{Rx}^{i-1} = \mathbf{F}_{res}\mathbf{S}_{Rx-BF}^H$ |
| $\quad q = arg\ max_{m=1\ldots M}(\mathbf{\Psi}_{Tx}^{i-1}(\mathbf{\Psi}_{TTx}^{i-1})^H)_{m,m}$ |
| $\quad \overline{\mathbf{F}}_{Rx} = [\overline{\mathbf{F}}_{Rx}^T \vert (\mathbf{S}_{Rx-BF}^T)(:,k)]$ |
| $\quad \mathbf{\Psi}_{Tx}^{i-1} = \mathbf{S}_{Tx-BF}^H\mathbf{F}_{res}\overline{\mathbf{F}}_{Rx}^H$ |
| $\quad l = arg\ max_{m=1\ldots M}(\mathbf{\Psi}_{Rx}^{i-1}(\mathbf{\Psi}_{Rx}^{i-1})^H)_{m,m}$ |
| $\quad \overline{\mathbf{F}}_{Tx} = [\mathbf{F}_{Tx-RF} \vert \mathbf{S}_{Tx-BF}(:,k)]$ |
| $\quad \underline{\mathbf{F}} = (\overline{\mathbf{F}}_{Tx}^H\overline{\mathbf{F}}_{Tx})^{-1}\overline{\mathbf{F}}_{Tx}^H\mathbf{F}^o \overline{\mathbf{F}}_{Rx}^H(\overline{\mathbf{F}}_{Rx}\overline{\mathbf{F}}_{Rx}^H)$ |
| $\quad \mathbf{F}_{res} = \frac{\mathbf{F}^o - \overline{\mathbf{F}}_{Tx}\underline{\mathbf{F}}\overline{\mathbf{F}}_{Rx}}{\vert\vert\mathbf{F}^o - \overline{\mathbf{F}}_{Tx}\underline{\mathbf{F}}\overline{\mathbf{F}}_{Rx}\vert\vert_F}$ |
| end for |
| return $\overline{\mathbf{F}}_{Tx}, \overline{\mathbf{F}}_{Rx}$, and$\underline{\mathbf{F}}$ |

*2) Hybrid OMP based Relay Filter design:* The optimal relay gain matrix obtained in (20) is decomposed into hybrid matices such that $||\mathbf{F}^o - \overline{\mathbf{F}}_{Tx}\underline{\mathbf{F}}\overline{\mathbf{F}}_{Rx}||_F$ is minimized. Following the approach in [12], OMP sparse problem in this case can be formulated as,

$$(\overline{\mathbf{F}}_{Tx}^o, \underline{\mathbf{F}}^o, \overline{\mathbf{F}}_{Rx}^o) = \arg \min_{\overline{\mathbf{F}}_{Tx}, \underline{\mathbf{F}}, \overline{\mathbf{F}}_{Rx}} ||\mathbf{F}^o - \overline{\mathbf{F}}_{Tx}\underline{\mathbf{F}}\overline{\mathbf{F}}_{Rx}||_F$$

$$\text{s.t.} \qquad \overline{\mathbf{F}}_{Tx}^{(j)} \in \mathbf{S}_{Tx-BF}, (\overline{\mathbf{F}}_{Rx})^{(j)} \in \mathbf{S}_{Rx-BF}, and$$

$$||\overline{\mathbf{F}}_{Tx}\underline{\mathbf{F}}\overline{\mathbf{F}}_{Rx}||_F^2 = P_r. \tag{28}$$

To solve this optimization problem, the RF beamforming matrices for transmitting and receiving signals from the users $\overline{\mathbf{F}}_{Tx}$ and $\overline{\mathbf{F}}_{Rx}$ respectively are jointly selected from the set of candidate vectors $\mathbf{S}_{Tx-BF}$ and $\mathbf{S}_{Rx-BF}$. When both the vectors are selected the effective residue is updated at each step, and corresponding baseband filter is obtained similarly by least square solution. Hence, the overall error is minimized with each iteration. The complete OMP algorithm for the obtaining hybrid relay processor is given in Table III.

### *C. Robust Hybrid Two-way AF Relay based mmWave System Design*

In the previous subsection, we designed the system considering that all the processing units possess perfect CSI knowledge. While, imperfections can be introduced in the CSI due to various reasons [14] and affect the overall performance of the system. Hence, we extend the above proposed design to a robust system by considering imperfections in CSI. We proceed with the similar approach as discussed in previous subsections III-A and III-B by adding the imperfections to channels. However in order to incorporate the imperfections in the available channel knowledge, we consider the channel model given in (6) during the the overall system design. This new channel gain is modeled by assuming that the errors in the channel are distributed as Gaussian random vectors with known error variance. Further, following the similar approach, we jointly design optimal filters $\mathbf{V}_i^o$, $\mathbf{F}^o$ and $\mathbf{R}_i^o$ by considering expected values of the optimization objective and constraints. The expectation here, is with respect to the CSI error variables, thus proving immunity to the performance degradation result-

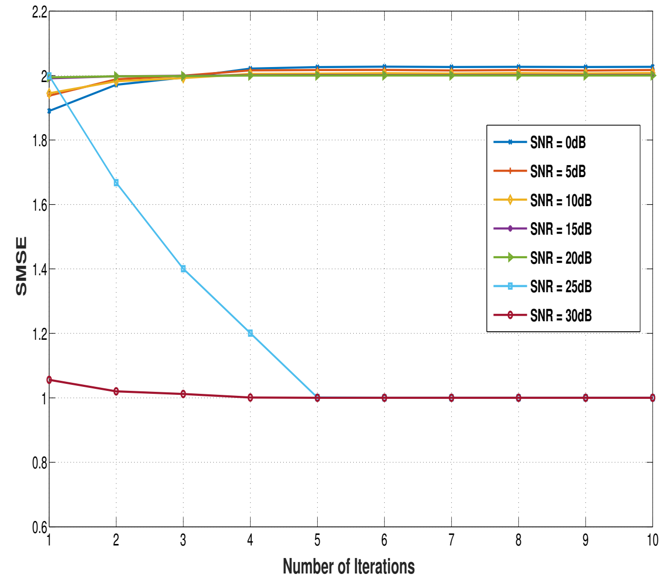


Fig. 2. MSE Convergence plot for the proposed algorithm

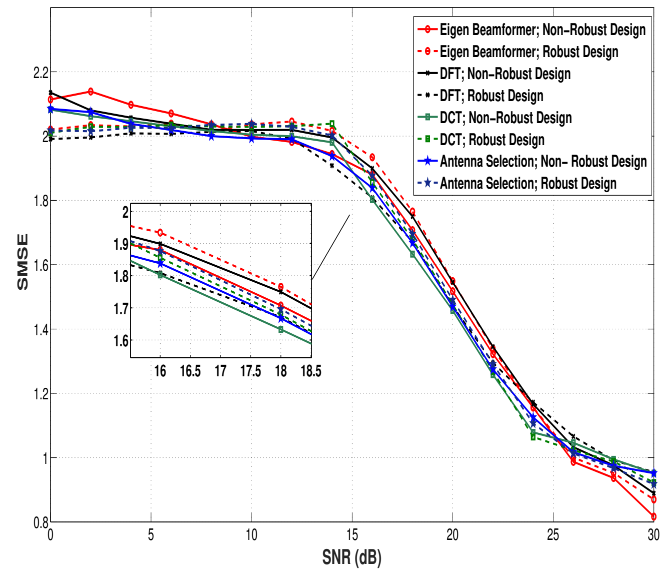


Fig. 4. SMSE for Non-robust (solid line) and robust (dashed line) hybrid system against varying SNR for multiple dictionaries

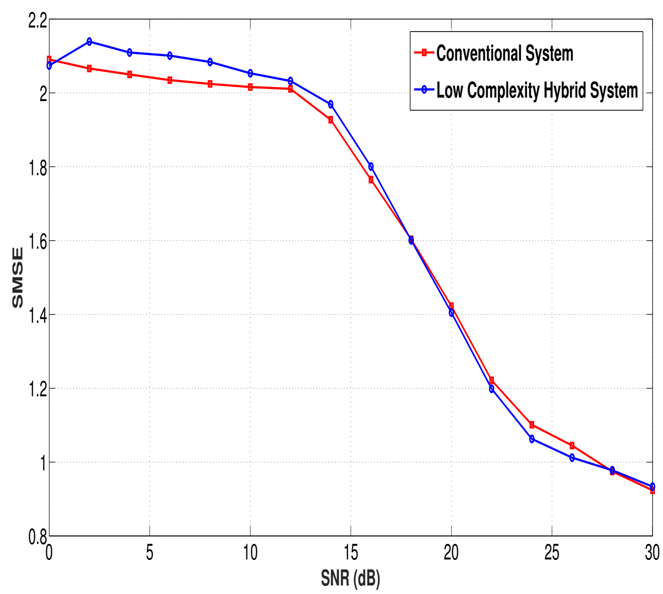


Fig. 3. SMSE for conventional (solid line) and Hybrid (dashed line) system against varying SNR

ing form the CSI errors. We further decompose and obtain the hybrid filters in the similar way by using the OMP algorithm given in Table II and III. The comparison results for both the proposed designs is given in simulation results.

## IV. Simulation Results

This section presents the simulation results for proposed non-regenerative HD two way relay assisted mmWave communication systems. In particular, we compare the performance of both the proposed robust and non robust design with each other and with conventional two-way MIMO relay from the literature. Simulations are performed for $Nt = 6$, $Nr_{Tx} = Nr_{Rx} = 16$, and with number of RF chains $N_{RF} = 3$ for the users and $Nr_{RF} = 14$ for processing at relay. The channels are quasi-static flat fading Rayleigh channel with stochastic errors in CSI modeled as Gaussian random vector. It is observed from the results that the proposed hybrid system converges to an optimal point in maximum of 5 iterations as shown Figure 2. Even though global convergence is hard to prove due to the non convex nature of optimization problem, the convergence of algorithm to a limit is guaranteed and resulting in comparable SMSE values to that of conventional digital MIMO relay system. The comparison results for SMSE plotted against varying SNR for proposed hybrid system and conventional MIMO system is shown in Figure 3. For simulation results in Figure 3, we considered the eigen beamformers for hybrid RF design and it is observed that reduced hardware complexity is achieved without compromising on the performance with the hybrid design. In order to form RF beamforming matrix for hybrid decomposition, we consider various dictionaries from which the candidate beamforming vectors are selected. These dictionaries includes, eigen beamforming, discrete fourier transform (DFT), discrete cosine transform (DCT) and antenna selection beamforming. The performance have been evaluated over all these dictionaries for both the proposed schemes. SMSE and sum rate plots are shown in Figure 4 and 5 respectively considering above dictionaries. It has been observed that SMSE decreases with increasing SNR, however there is no significant difference in the error for different dictionaries. Similar behavior is observed in results for sum-rate in Figure 5, where higher sum-rate is obtained with increasing SNR. Hence, even though eigen beamformer performs best among all, any of the above mentioned dictionary can be selected, where selection may vary according to the application and implementation simplicity.

## V. Conclusion

This paper investigated hybrid MIMO processors for non-regenerative two-way AF relay-assisted mmWave communi-

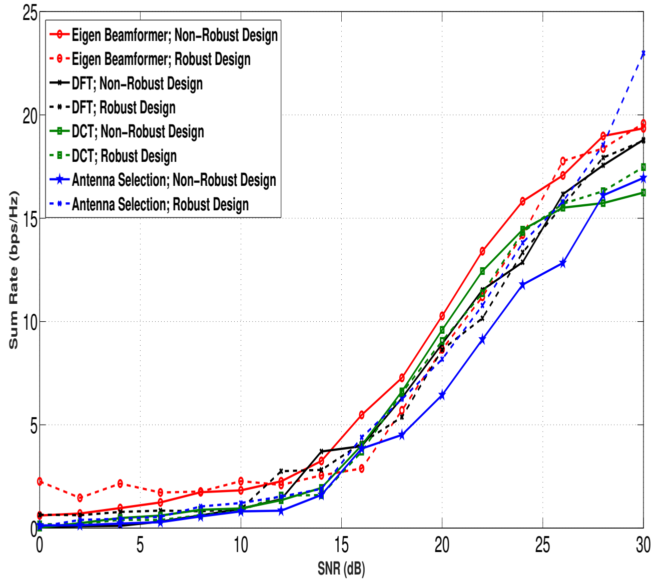


Fig. 5. Sum-rate for non-robust (solid line) and robust (dashed line) hybrid system against varying SNR for multiple dictionaries

cation system. We proposed joint design for optimal precoder, receive filter and AF relay by minimizing SMSE under the constraint on total relay transmit power. These jointly obtained optimal filters were further decomposed into analog-digital hybrid processors by OMP based sparse signal processing technique and achieved low complexity system design. The design was developed by assuming perfect CSI knowledge and was also extended to a robust design by assuming erroneous CSI. The performance of both the designs was evaluated on various parameters and was demonstrated with simulation results. We showed that the proposed algorithm converges to a limit. The performance for both the proposed designs was tested for various dictionaries including eigen beamforming, DCT, DFT and antenna selection. It has been observed that eigen beamformer performs best among all. We also compared robust and non robust design and results show that in the presence of CSI errors, the robust design perfroms better as it was resilient to the erroneous CSI.

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