Energy-Efficient Hybrid Transceiver Designs for Millimeter Wave Communication Systems

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***Abstract*—In this paper, we consider the design of low-complexity analog-digital hybrid transceivers for a multi-input multi-output (MIMO) millimeter wave (mmWave) communication system operating in a $K-$user interference channel. Our objective is to achieve the energy efficiency by minimizing overall transmit power required for achieving a target mean-square error (MSE) at the receivers. We propose two transceiver designs based on the quality of the available channel state information (CSI). We first present a design that minimizes total transmit power assuming the availability of perfect channel knowledge. Later, we extend it to a robust transceiver design by considering imperfections in CSI. In this case, error in the available CSI is modeled as Gaussian random variables. The proposed designs achieve reduced hardware and computational complexity by forming hybrid architecture using sparse signal processing. Hence, achieving similar MSE with lower energy consumption. We evaluate the performance of both the proposed schemes based on various parameters. Furthermore, we demonstrate the resilience of the robust design in presence of errors in CSI and performance of both the designs with various dictionaries.**

***Keywords*—mmWave communication, sparse signal processing, hybrid transceiver, orthogonal matching pursuit, 5G.**

I. Introduction

Millimeter wave (mmWave) communication is expected to be an inevitable component of the suite of technologies for next generation cellular systems, harnessing which opens up availability of abundant spectral resource [1], [2]. The potential benefits are however accompanied by various challenges such as increased free space path loss, less significant scattering, pronounced coverage and blockage holes [3]. Some of these challenges can be overcome by high-gain electronically steerable directional antennas that provide high signal-noise-ratio (SNR) by beamforming or precoding data on large-scale antenna arrays, making multiple-input multiple-output (MIMO) a key technique for mmWave systems. Conventional full-complexity MIMO schemes use one radio frequency (RF) chain per antenna. However, dedicating a separate RF chain for each antenna will lead to high power consumption in mmWave systems. It is thus imperative to reduce the power consumption by reducing number of RF chains compared to the number of antenna elements in order to make the system less complex and amenable to practical implementation.

Hybrid architecture is a promising solution to reduce the hardware complexity in mmWave systems [4]. Hybrid processors processes data in two sequential phases, *viz.*, digital baseband processing followed by analog beamforming [3]. Signal combining in analog domain achieves the reduction in number of RF chains. Some recent results point to the effectiveness of such techniques [5], [6]. In this paper, we propose to develop energy efficient and practically realizable hybrid transceivers using sparse signal processing technique. Specifically, we adopt orthogonal matching pursuit (OMP) algorithm to achieve significant reduction in hardware complexity [7]. Another factor that affects the system performance in general is the quality of the available channel state information (CSI). Various factors such as feedback delays, estimation error, pilot contamination in the case of large MIMO, can introduce errors in CSI estimation. Systems developed assuming the perfect knowledge of CSI are likely to suffer performance degradation in the presence of such CSI errors [8]. Thus in practice, the systems that are resilient to such errors are highly desirable.

In this paper, we propose two hybrid transceiver designs with reduced hardware complexity for multi-user interference channel. We first introduce a low complexity mmWave transceiver design that assumes the availability of perfect channel knowledge. This will be referred to as non-robust design in rest of the paper. Later, we propose a robust design by considering the imperfections in the CSI. This design makes use of suitably modified performance metrics to combat the effect of errors in available CSI on the system performance. In each of the proposed designs, we first jointly design full-digital optimal precoder and receive filters, by minimizing total transmit power. Resultant optimal filters have high hardware and computational complexity due to large number of antennas in mmWave systems. However, we partition the overall processing to digital baseband and analog radio frequency (RF) processing by using sparse approximation. This partition makes reduced-complexity implementation possible. The corresponding optimization problem is not a convex problem and hence the global solution is not guaranteed. However, we show that proposed solution converges to an optimal point. We carry out the performance evaluation of both the schemes with extensive simulations over various parameter values and demonstrate the comparison results later in the paper. The rest of this paper is organized as follows. System and channel model is described in Section II. The proposed low-complexity hybrid non-robust and robust transceiver designs are discussed in Section III. Section IV presents the simulation results. Finally, Section V concludes this paper.

Notations: Throughout this paper, we use bold-faced lower-case letters to denote column vectors and bold-faced uppercase letters to denote matrices. $\underline{\mathbf{X}}$ and $\overline{\mathbf{X}}$ implies that the variable $\mathbf{X}$ corresponds to the baseband and RF block, respectively. $\mathrm{tr}(\cdot)$, $\mathbb{E}\{\cdot\}$, $\|\cdot\|_0$ and $\|\cdot\|_F$ denotes the trace operator, expectation operator, 0-norm and Frobenius-norm respectively.

II. System and Channel Model

A. System Model

We consider a $K-$user interference channel as shown in Fig. 1, where transmitters and receivers are equipped with analog-digital hybrid precoders and combiners, respectively. Each transmitter transmits N_S symbols over nTx antennas and each receiver is equipped with nRx antennas. The signal transmitted by the kth transmitter is denoted by the N_s-dimensional column vector $\mathbf{d}_k$. The number of RF chains associated with each transmitter and receiver are $\overline{N}_t$ and $\overline{N}_r$, respectively, and $N_s \leq \overline{N}_t < nTx$ and $N_s \leq \overline{N}_r < nRx$. The $\overline{N}_t \times N_s$ matrix $\underline{\mathbf{V}}_k$ denotes the digital baseband precoder and the $N_t \times \overline{N}_t$ matrix $\overline{\mathbf{V}}_k$ denotes the RF beamformer at the kth transmitter. On the receiver side, the signal vector received by the k^{th} receiver is passed through a RF beamfomer denoted by $nRx \times \overline{N}_r$ matrix $\overline{\mathbf{R}}_k$ followed by a baseband combiner denoted by the $\overline{N}_r \times N_s$ matrix $\underline{\mathbf{R}}_k$. The output at the k^{th} RF beamformer is $\mathbf{z}_k$, where, $\mathbf{z}_k = \overline{\mathbf{R}}_k^H \mathbf{y}_k$. Let $\mathbf{V}_k^o$ and $\mathbf{R}_k^o$ denote the optimal precoder and receive filter in the conventional MIMO interference channel. Then, we design the hybrid filters to satisfy the following: $\mathbf{V}_k^o = \overline{\mathbf{V}}_k \underline{\mathbf{V}}_k$ and $\mathbf{R}_k^o = \underline{\mathbf{R}}_k^H \overline{\mathbf{R}}_k^H$. Following the above system model, the estimate of the transmitted data can be given by,

$$\widehat{\mathbf{d}}_k = \underline{\mathbf{R}}_k^H \mathbf{z}_k. \tag{1}$$

B. Channel Model

Due to high free-space pathloss and the use of large tightly-packed antenna at mmWave frequencies, a 2D narrowband parametric clustered channel model is adopted. We consider extended Saleh-Valenzuela model [9], in which the channel matrix $\mathbf{C}_{kj}$, from j^{th} transmitter to k^{th} receiver can be characterized as follows,

$$\mathbf{C}_{kj} = \gamma \sum_{m=1}^{N_{cl}} \sum_{n=1}^{N_{ray}} \alpha_{mn}^{kj} \mathbf{a}_r\big(\phi_{mn}^{r(k)}, \theta_{mn}^{r(k)}\big) \mathbf{a}_t\big(\phi_{mn}^{t(j)}, \theta_{mn}^{t(j)}\big), \tag{2}$$

where, N_{ray} is the number of rays in N_{cl} clusters and, the normalization factor $\gamma = \sqrt{\frac{nTxnRx}{N_{cl}N_{ray}}}$ is such that it satisfies $\mathrm{E}\big[||\mathbf{C}_{\mathrm{kj}}{}_{\mathrm{F}}^2||\big] = \mathrm{nTx} \times \mathrm{nRx}$. α_{mn} denotes the complex gain of n^{th} ray in m^{th} cluster and is assumed to be i.i.d. and complex Gaussian random variables with zero mean and variance $\sigma_\alpha^2 \sim \mathcal{N}(0, \sigma_\alpha^2)$. $\mathbf{a}_t\big(\phi_{mn}^{t(j)}, \theta_{mn}^{t(j)}\big)$ and $\mathbf{a}_r\big(\phi_{mn}^{r(k)}, \theta_{mn}^{r(k)}\big)$ are the array response vectors at the transmitter and receiver respectively, and $\phi_{mn}^{t(j)}, \theta_{mn}^{t(j)}$ and $\phi_{mn}^{r(j)}, \theta_{mn}^{r(j)}$ are azimuthal and elevation angle for transmit and receive antennas respectively, where,

$$\mathbf{a}\big(\phi_{mn}, \theta_{mn}\big) = \frac{1}{\sqrt{nTx}} \big[\exp^{\iota m \times \frac{2\pi}{\lambda} \mathrm{d}(\sin(\phi))}\big]^T. \tag{3}$$

We assume the transmitters possess only imperfect knowledge of the channel state, thus, the actual CSI can be modelled as,

$$\mathbf{C} = \widehat{\mathbf{C}} + \boldsymbol{\Delta}, \tag{4}$$

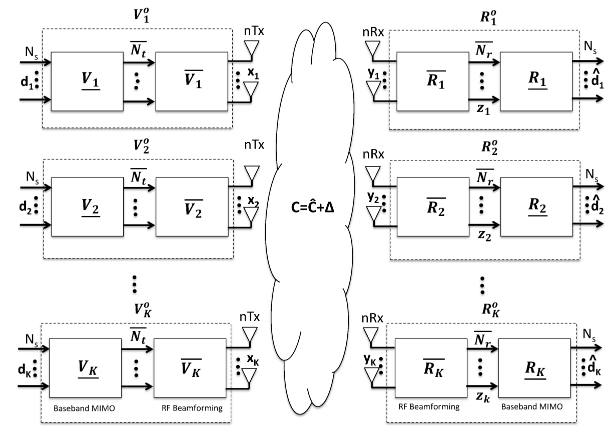

Fig. 1. mmWave Hybrid MIMO processor with K user interference channel.

where $\widehat{\mathbf{C}}$ is the estimated CSI available and $\boldsymbol{\Delta} \sim \mathcal{N}(0, \sigma_E^2)$ denote the corresponding error in the CSI. The additive noise at the receivers is white Gaussian noise, i.e., $\mathbf{n}_k \in \mathbb{C}^{nRx \text{ x } 1}$ with $\mathbf{n}_k \sim \mathcal{CN}(0, \sigma^2 \mathbb{I}_{nRx})$ for $k = 1, 2, \cdots, K$.

III. low-complexity Hybrid Transceiver Designs

In this section, we present the designs of hybrid precoding matrices $\overline{\mathbf{V}}_k$ and $\underline{\mathbf{V}}_k$ and hybrid receive filter matrices $\overline{\mathbf{R}}_k$ and $\underline{\mathbf{R}}_k$ for multiuser MIMO interference channel under perfect and imperfect channel state information. The proposed designs minimize total transmit power required to achieve a specified MSE at each receiver. Solving the corresponding optimization problem, we obtain the optimal full-complexity precoding and receive filter matrices, $\mathbf{V}_k^o$ and $\mathbf{R}_k^o$, respectively. Full-complexity design is based on the use of one RF chain per antenna. Subsequently, we obtain the low complexity hybrid precoders and receive filters from the optimal matrices using the OMP-based sparse approximation technique.

A. Full-Complexity Precoder and Receive Filter Design

We aim to design a set of optimal transmit precoder and receive filter matrices $\{\mathbf{V}_k^o, \mathbf{R}_k^o\}$, where $k = 1...K$, to minimize total transmit power under individual MSE constraint. Based on (1), the MSE at the k^{th} user is given by,

$$\begin{aligned}
\mathrm{MSE}_k &= \mathbb{E}[\|\widehat{\mathbf{d}}_k - \mathbf{d}_k\|^2] \\
&= \mathbb{E}\Big[\mathrm{tr}\Big(\Big(\big(\mathbf{R}_K(\widehat{\mathbf{C}}_{kk} + \alpha\boldsymbol{\Delta})\mathbf{V}_k - \mathbf{I}\big)\mathbf{d}_k \\
&+ \mathbf{R}_k \sum_{i=1}^{K} (\widehat{\mathbf{C}}_{ki} + \alpha\boldsymbol{\Delta})\mathbf{V}_i\mathbf{d}_i + \mathbf{R}_k\mathbf{n}_k\Big)\Big(\big(\mathbf{R}_K(\widehat{\mathbf{C}}_{kk} + \alpha\boldsymbol{\Delta})\mathbf{V}_k - \mathbf{I}\big)\mathbf{d}_k \\
&+ \mathbf{R}_k \sum_{i=1}^{K} (\widehat{\mathbf{C}}_{ki} + \alpha\boldsymbol{\Delta})\mathbf{V}_i\mathbf{d}_i + \mathbf{R}_k\mathbf{n}_k\Big)^H\Big)\Big] \\
&= \mathrm{tr}\Big(\mathbf{R}_k \sum_{i=1}^{K} (\widehat{\mathbf{C}}_{ki}\mathbf{V}_i\mathbf{V}_i^H\widehat{\mathbf{C}}_{ki}^H)\mathbf{R}_k^H - (\mathbf{R}_k\widehat{\mathbf{C}}_{kk}\mathbf{V}_k + \mathbf{V}_k^H\widehat{\mathbf{C}}_{kk}^H\mathbf{R}_k^H) \\
&+ \sigma_n^2\mathbf{R}_k\mathbf{R}_k^H + \mathbf{I}\Big) + \mathbb{E}\Big[\mathrm{tr}\Big(\mathbf{R}_\mathrm{k}\Big(\sum_{\mathrm{i}=1}^{\mathrm{K}} (\alpha\boldsymbol{\Delta})\mathbf{V}_\mathrm{i}\mathbf{V}_\mathrm{i}^\mathrm{H}(\alpha\boldsymbol{\Delta}^\mathrm{H})\Big)\mathbf{R}_\mathrm{k}^\mathrm{H}\Big)\Big].
\end{aligned} \tag{5}$$

The above equation is a generalized representation for MSE such that for $\alpha = 0$ and $\alpha = 1$, it represents the MSE for perfect and erroneousness CSI respectively. In order to further simplify (5), we use a Lemma in [8], which states that for any

TABLE I

Iterative algorithm computing $\mathbf{V}^o$ and $\mathbf{R}^o$ for power optimization
1. Initialize n = 0, $\mathbf{V}_k(0) \quad \forall k \in \{1,..K\}$ 2. Update $\mathbf{R}_k(n+1)$ using $\mathbf{V}_k(n)$, 3. Solve for λ_k: $\mathbf{V}_k(\lambda_k^-) = \Big(\mathbf{I} + \lambda_k \sum_{i=1}^{K} \widehat{\mathbf{C}}_{ik}^H \mathbf{R}_i^H(n+1)\mathbf{R}_i(n+1)\widehat{\mathbf{C}}_{ik}\Big)^{-1}$ $\times\Big(\lambda_k \widehat{\mathbf{C}}_{kk}^H \mathbf{R}_k^H(n+1) - \alpha(\sigma_E^2 \sum_{i=1}^{K} \mathrm{tr}(\mathbf{R}_k^H(n+1)\mathbf{R}_k(n+1)))\Big)$ $\lambda_k(n+1) = \Big[\{\tilde{\lambda_k} \mid \text{such that } \mathrm{MSE}_k = \mathrm{T}\}\Big]_+$ 4. Update $\mathbf{V}_k(n+1)$ using $\mathbf{R}_k(n+1)$ and $\lambda_k(n+1)$, 5. Repeat 2,3,4 until convergence. Non-Robust System : $\alpha = 0$; Robust System: $\alpha = 1$

random matrix $\mathbf{X}$ with $\mathbb{E}\{\mathbf{X}\mathbf{X}^H\} = \sigma^2\mathbf{I}$, and matrices $\mathbf{U}$ and $\mathbf{Z}$ of appropriate dimensions, the following equality holds:

$$\mathbb{E}[tr(\mathbf{X}\mathbf{U}\mathbf{X}^H\mathbf{Z})] = \mathbb{E}[tr(\mathbf{X}^H\mathbf{Z}\mathbf{X}\mathbf{U})] = \sigma^2 tr(\mathbf{U})tr(\mathbf{Z}). \tag{6}$$

Employing above result, we can simplify last term in (5) as,

$$\mathbb{E}\Big\{\mathrm{tr}\Big(\sum_{i=1}^{K} \mathbf{R}_k(\alpha\mathbf{\Delta})\mathbf{V}_i\mathbf{V}_i^H(\alpha\mathbf{\Delta}^H)\mathbf{R}_k^H\Big)\Big\} = \alpha\sum_{i=1}^{K}\sigma_E^2\mathrm{tr}(\mathbf{V}_i\mathbf{V}_i^H)\mathrm{tr}(\mathbf{R}_k^H\mathbf{R}_K). \tag{7}$$

Based on the preceding developments, the sum-MSE for a $K-$user interference channel can be expressed as,

$$\sum_{k=1}^{K}\mathrm{MSE_k} = \mathrm{tr}\Big(\sum_{k=1}^{K}\mathbf{R}_k\sum_{i=1}^{K}\widehat{\mathbf{C}}_{ki}\mathbf{V}_i\mathbf{V}_i^H\widehat{\mathbf{C}}_{ki}^H\mathbf{R}_k^H - \sum_{k=1}^{K}(\mathbf{R}_k\widehat{\mathbf{C}}_{kk}\mathbf{V}_k + \mathbf{V}_k^H\widehat{\mathbf{C}}_{kk}^H\mathbf{R}_k^H) + \sum_{k=1}^{K}\sigma_n^2\mathbf{R}_k\mathbf{R}_k^H + K\mathbf{I}\Big) + \alpha\sum_{k=1}^{K}\sum_{i=1}^{K}\sigma_E^2\mathrm{tr}(\mathbf{V}_i\mathbf{V}_i^H)\mathrm{tr}(\mathbf{R}_k^H\mathbf{R}_k). \tag{8}$$

We design optimal transceivers by minimizing the total transmit power under constraints on MSE at each user. Hence, the optimization problem can be mathematically expressed as,

$$\min_{\{\mathbf{V_k}\},\{\mathbf{R_k}\}} \quad \sum_{k=1}^{K}\mathrm{tr}(\mathbf{V}_k^H\mathbf{V}_k) \tag{9}$$
$$\text{subject to:} \quad \mathrm{MSE}_k \le T_k, \forall k \in \{1..K\},$$

where MSE_k is given as in equation (8) and T_k is the upper bound on MSE for the k^{th} user. This optimization problem can be solved using the Karush-Kuhn-Tucker conditions to be satisfied by the optimal solution. The Langrangian associated with (9) is given by,

$$L(V_k, R_k, \lambda_k) = \sum_{k=1}^{K}\mathrm{tr}(\mathrm{V_k^H V_k}) + \sum_{\mathrm{k=1}}^{\mathrm{K}}\lambda_\mathrm{k}[\mathrm{MSE_k} - \mathrm{T_k}], \tag{10}$$

where λ_k, $k = 1, 2, \cdots, K$ are Lagrangian variables.

It can be observed that the formulated problem is not jointly convex in the optimization variables, but it is convex in $\{\mathbf{V}_k\}$ for fixed values of $\{\mathbf{R}_k\}$ and *vice verse*. Based on this, we obtain a solution by coordinate descent method, wherein the minimization is performed w.r.t. one variable while keeping other variables fixed. Thus, the optimal values $\mathbf{V}^o$ and $\mathbf{R}^o$ are obtained iteratively. Solving (9) based on different available CSI, two corresponding transceiver designs are developed and the details are discussed in subsequent subsections.

1) Non-Robust Design: In this section, we assume that the CSI is globally and perfectly available. We jointly design optimal filters, by considering expected values of the optimization objective and constraints. For the design of non-robust transceiver, we set $\alpha = 0$ in (8). Solving (10), the Lagrangian associated with non-robust transceiver design can be given as,

$$L = \sum_{k=1}^{K} tr(\mathbf{V}_k\mathbf{V}_k^H) + \sum_{k=1}^{K}\lambda_k\Big\{tr\big[\mathbf{R}_k(\sum_{i=1}^{K}\widehat{\mathbf{C}}_{ki}\mathbf{V}_i\mathbf{V}_i^H\widehat{\mathbf{C}}_{ki}^H)\mathbf{R}_k^H - (\mathbf{R}_k\widehat{\mathbf{C}}_{kk}\mathbf{V}_k + \mathbf{V}_k^H\widehat{\mathbf{C}}_{kk}^H\mathbf{R}_k^H) + \mathbf{I} + \sigma_n^2\mathbf{R}_k\mathbf{R}_k^H\big]\Big\}. \tag{11}$$

Differentiating it w.r.t. $\mathbf{V}_k^*$, and $\mathbf{R}_k^*$ respectively and equating it to 0, we obtain the corresponding full-complexity precoding and receive filter matrices as follows,

$$\frac{\partial L}{\partial \mathbf{V}_k^*} = 0, \qquad \forall k \in (1...K). \tag{12}$$

Thus, the precoding matrix for a given $\mathbf{R}_k$ can be given by,

$$\mathbf{V}_k^o = (\mathbf{I} + \lambda_k\sum_{i=1}^{K}\widehat{\mathbf{C}}_{ik}^H\mathbf{R}_i^H\mathbf{R}_i\widehat{\mathbf{C}}_{ik})^{-1}\widehat{\mathbf{C}}_{kk}^H\mathbf{R}_k^H. \tag{13}$$

And,

$$\frac{\partial L}{\partial \mathbf{R}_k^*} = 0, \qquad \forall k \in (1...K). \tag{14}$$

Hence, the receive filter matrix for given $\mathbf{V}_k$ can be given as,

$$\mathbf{R}_k^o = \mathbf{V}_k^H\widehat{\mathbf{C}}_{kk}^H(\sum_{i=1}^{K}\widehat{\mathbf{C}}_{ki}\mathbf{V}_i\mathbf{V}_i^H\widehat{\mathbf{C}}_{ki}^H + \sigma_n^2 I)^{-1}. \tag{15}$$

2) Robust Design: The channel model considered for this design is as given in (4). In other words, we design the system by considering the imperfections in the available channel knowledge. Following the similar approach as in the previous subsection dealing with the non-robust transceiver design, we obtain the solutions for robust optimal precoding and receive filters. In order to make the performance immune to errors in the available CSI, we consider the expected value of the performance matrix in optimization problem. Formulating and solving the new optimization problem for the robust design, expressions for full-complexity optimal $\mathbf{V_k^o}$ and $\mathbf{R_k^o}$ are as given below,

$$\mathbf{V}_k^o = (\mathbf{I} + \lambda_k\sum_{i=1}^{K}\widehat{\mathbf{C}}_{ik}^H\mathbf{R}_i^H\mathbf{R}_i\widehat{\mathbf{C}}_{ik})^{-1}(\widehat{\mathbf{C}}_{kk}^H\mathbf{R}_k^H - \sigma_E^2\sum_{i=1}^{K}\mathrm{tr}(\mathbf{R}_k^H\mathbf{R}_k)), \tag{16}$$

$$\mathbf{R}_k^o = (\mathbf{V}_k^H\widehat{\mathbf{C}}_{kk}^H - \sigma_E^2\sum_{i=1}^{K}\mathbf{V}_i\mathbf{V}_i^H)(\sum_{i=1}^{K}\widehat{\mathbf{C}}_{ki}\mathbf{V}_i\mathbf{V}_i^H\widehat{\mathbf{C}}_{ki}^H + \sigma_n^2\mathbf{I})^{-1}. \tag{17}$$

The generalized iterative algorithm for both the designs obtained in subsections 1 and 2 are given in Table-I. Since the objective function i.e., total transmit power, is non-increasing with each iteration, and it is lower-bounded, the objective function tends to limit as the number of iteration increases.

B. Hybrid OMP-Based Precoder and Receive Filter Design

Orthogonal Matching Pursuit (OMP) for sparse approximation is a greedy approach that iteratively updates the initial zero estimates by minimizing the approximation cost w.r.t. all the currently selected coefficients. OMP is a well known signal processing algorithm, and has been extensively studied in literature for various applications [10]–[12]. As described earlier, it leads to reduced complexity implementation for mmWave systems. We use the OMP sparse approximation technique in order to decompose the optimal full-complexity matrices $\{\mathbf{V}_k^o\}$ and $\{\mathbf{R}_k^o\}$, for non-robust design in (13), (15) and for robust design in (16), (17), into their corresponding baseband and RF processing matrices. The baseband and RF matrices for both the designs are realized by decomposing optimal solution as $\mathbf{V}_k^o = \overline{\mathbf{V}}_k \underline{\mathbf{V}}_k$ and $\mathbf{R}_k^o = \underline{\mathbf{R}}_k^H \overline{\mathbf{R}}_k^H$. A generalized detailed OMP based iterative algorithm discussing the computation steps for jointly designing the RF and baseband hybrid filters at Tx and Rx is given in Table.II. The details of this design are discussed in following subsections.

1) Design of Hybrid Receive Filters in MIMO Systems: In this section, we seek to design hybrid combiners $\overline{\mathbf{R}}_\mathbf{k}, \underline{\mathbf{R}}_\mathbf{k}$ that minimizes residue between optimal and decomposed matrices. Let, $\mathbf{\Gamma}_{\mathbf{y}_k} = \mathbb{E}[\mathbf{y}_k \mathbf{y}_k^H]$ and $\mathbf{\Gamma}_{\mathbf{y}_k \widehat{\mathbf{d}}_k} = \mathbb{E}[\mathbf{y}_k \widehat{\mathbf{d}}_k]$, where, $\widehat{\mathbf{d}}_k = \mathbf{R^o}^H \mathbf{y}_k$. Thus,

$$\mathbf{\Gamma}_{\mathbf{y}_k \widehat{\mathbf{d}}_k} = \mathbb{E}[\mathbf{y}_k \mathbf{y}_k^H \mathbf{R}^o], \tag{18}$$

$$\mathbf{\Gamma}_{\mathbf{y}_k \widehat{\mathbf{d}}_k} = \mathbf{\Gamma}_{\mathbf{y}_k} \mathbf{R}^o. \tag{19}$$

Hence, for the receiver design problem, the optimal solution in (15) or (17) by considering the respective channel knowledge can be rewritten as,

$$\mathbf{R}_k^o = \mathbf{\Gamma}_{\mathbf{y}_k}^{-1} \mathbf{\Gamma}_{\mathbf{y}_k \widehat{\mathbf{d}}_k}, \tag{20}$$

Similarly, the baseband receive filter matrix can be given by,

$$\underline{\mathbf{R}}_k^o = \mathbf{\Gamma}_{\mathbf{z}_k}^{-1} \mathbf{\Gamma}_{\mathbf{z}_k \widehat{\mathbf{d}}_k} = (\overline{\mathbf{R}}_k^H \mathbf{\Gamma}_{\mathbf{y}_k} \overline{\mathbf{R}}_k)^{-1} \overline{\mathbf{R}}_k^H \mathbf{\Gamma}_{\mathbf{y}_k \widehat{\mathbf{d}}_k}, \tag{21}$$

where $\mathbf{\Gamma}_{\mathbf{z}_k} = \mathbb{E}[\mathbf{z}_k \mathbf{y}_k^H]$ and $\mathbf{\Gamma}_{\mathbf{z}_k \widehat{\mathbf{d}}_k} = \mathbb{E}[\mathbf{z}_k \widehat{\mathbf{d}}_k^H]$. The OMP sparse problem for receiver can be formulated as,

$$\overline{\mathbf{R}}_k^o = \underset{\overline{\mathbf{R}}_k}{\operatorname{argmin}} \, \mathbb{E} \| \mathbf{d}_k - \underline{\mathbf{R}}_k^{oH} \overline{\mathbf{R}}_k \mathbf{y}_k \|^2, \tag{22}$$

Equation (22) is the standard form to which OMP algorithm can be applied, which can be rewritten as,

$$\overline{\mathbf{R}}_k^o = \underset{\overline{\mathbf{R}}_k^o}{\operatorname{argmin}} \, \| \mathbf{\Gamma}_{\mathbf{y}_k}^{\frac{1}{2}} \mathbf{R}_k^o - \mathbf{\Gamma}_{\mathbf{y}_k}^{\frac{1}{2}} \overline{\mathbf{R}}_k \underline{\mathbf{R}}_k^o \|_F^2. \tag{23}$$

Introducing a dictionary $\mathbf{S}_{BF}$, where basically dictionary constitutes a set of candidate beamforming vectors which can be used during RF design, the optimization problem can be rephrased as,

$$\tilde{\underline{\mathbf{R}}}_k^o = \underset{\tilde{\underline{\mathbf{R}}}_k}{\operatorname{argmin}} \, \| \mathbf{\Gamma}_{\mathbf{y}_k}^{\frac{1}{2}} \mathbf{R}_k^o - \mathbf{\Gamma}_{\mathbf{y}_k}^{\frac{1}{2}} \mathbf{S}_{BF} \tilde{\underline{\mathbf{R}}}_k^o \|_F^2, \tag{24}$$

$$\textit{s.t.} \ \| \text{diag} \ (\tilde{\underline{\mathbf{R}}}_k^o \tilde{\underline{\mathbf{R}}}_k^{oH}) \|_0 = \overline{N}_r.$$

Thus in (24), $\mathbf{\Gamma}_{\mathbf{y}_k}^{\frac{1}{2}} \mathbf{R}_k^o$ are approximated by the columns of $\mathbf{\Gamma}_{\mathbf{y}_k}^{\frac{1}{2}} \mathbf{S}_{BF}$ with non zero elements of $\tilde{\underline{\mathbf{R}}}_k^o$ as weights for the linear combination.

2) Design of Hybrid Precoders in MIMO Systems: In order to design hybrid precoder matrices, we consider the equivalent system for modeling reverse MIMO interference channel as shown in Fig.2. Considering the reverse system, optimal precoders of forward system can be obtained from optimal receive filters of the reverse system. Thus for any k^{th} user in reverse system, the transceivers are considered to have $\mathbf{R}_k$ and $\mathbf{V}_k$ as their precoding and receive filter units respectively. The interference channel is modeled as C_{ik}^H from i^{th} transmitter to k^{th} receiver for any $i \neq k$ and $\mathbf{n}_k^r$ is considered as the additive white Gaussian noise at the receiver $\mathbf{n}_k \in \mathbb{C}^{nTx \text{ x } 1}$. Thus the received signal for the reverse system $\mathbf{y_k^r}$ can be given by,

$$\mathbf{y}_k^r = \mathbf{C}_{kk}^H \mathbf{R}_k \mathbf{d}_k^r + \sum_{i \neq k}^{K} \mathbf{C}_{ik}^H \mathbf{R}_i \mathbf{d}_k^r + \mathbf{n}_k^r. \tag{25}$$

And the signal estimate for reverse channel can be given by,

$$\widehat{\mathbf{d}}_k^r = \underline{\mathbf{V}}_k^H \mathbf{y}_k^r. \tag{26}$$

Thus, similar to the hybrid receive filters design discussed in previous subsection, the optimization problem for $\overline{\mathbf{V}}_k^o$ can be written as,

$$\overline{\mathbf{V}}_k^o = \underset{\overline{\mathbf{V}}_k}{\operatorname{argmin}} \, \| \mathbf{\Gamma}_{\mathbf{y}_k^r}^{\frac{1}{2}} \mathbf{V}_k^o - \mathbf{\Gamma}_{y_k^r}^{\frac{1}{2}} \overline{\mathbf{V}}_k \underline{\mathbf{V}}_k^o \|_F^2, \tag{27}$$

where, $\mathbf{\Gamma}_{\mathbf{y}_k^r} = \mathbb{E}[\mathbf{y}_k^r \mathbf{y}_k^{rH}]$, $\mathbf{\Gamma}_{\mathbf{y}_k^r \widehat{\mathbf{d}}_k} = \mathbb{E}[\mathbf{y}_k^r \widehat{\mathbf{d}}_k^H]$, and the baseband receive filter matrix $\underline{\mathbf{V}}_k^o = (\overline{\mathbf{V}}_k \mathbf{\Gamma}_{\mathbf{y_k^r}} \overline{\mathbf{V}}_k)^{-1} \overline{\mathbf{V}}_k^H \mathbf{\Gamma}_{\mathbf{y}_k^r \widehat{\mathbf{d}}_k^r}$. Hence, the OMP sparse problem for hybrid precoders can be rewritten as,

$$\overline{\mathbf{V}}_k^o = \underset{\tilde{\mathbf{V}}_k}{\operatorname{argmin}} \, \| \mathbf{\Gamma}_{\mathbf{y}_k^r}^{\frac{1}{2}} \mathbf{V}_k^o - \mathbf{\Gamma}_{\mathbf{y}_k^r}^{\frac{1}{2}} \mathbf{S}_{BF} \tilde{\mathbf{V}}_k^o \|_F^2, \tag{28}$$

$$\text{s.t.} \ \| \text{diag} \ (\tilde{\underline{\mathbf{V}}}_k^o \tilde{\underline{\mathbf{V}}}_k^{oH}) \|_0 = \overline{N}_t \text{ and } \| \mathbf{S}_{BF} \tilde{\underline{\mathbf{V}}}_k \|_F^2 = \| \mathbf{V}_k^o \|_F^2.$$

where, the column of $\overline{V}_k$ are selected from set of candidate beamformers $\mathbf{S}_{BF}$.

To solve this optimization problem, the RF beamforming matrix is selected from the set of candidate vectors $\mathbf{S}_{BF}$. At each iteration, the effective residue is updated from formulated beamformer matrix, and corresponding baseband filter is obtained by least square solution. Hence, minimizing the overall error with each iteration as given in Table.II.

TABLE II

OMP-based iterative algorithm for robust MSE optimization

Require $\mathbf{P}_k^o, \mathbf{\Phi}, \mathbf{S}_{BF}$

$\overline{\mathbf{Q}}_k = [\ ]$

$\mathbf{A}_0 = \mathbf{P}_k^o$

for $i = 1$ to $\overline{N}$ do

$\quad \mathbf{\Psi}_{i-1} = (\mathbf{\Phi} \mathbf{S}_{BF})^H (\mathbf{\Phi} \mathbf{A}_{i-1})$

$\quad \underline{l = arg\ max}_{m=1 \ldots M} (\mathbf{\Psi}_{i-1} \mathbf{\Psi}_{i-1}^H)_{m,m}$

$\quad \overline{\mathbf{Q}}_k = [\overline{\mathbf{Q}}_k | \mathbf{\Phi}(:, k)]$

$\quad \underline{\mathbf{Q}}_k = (\overline{\mathbf{Q}}_k^H \overline{\mathbf{Q}}_k)^{-1} \overline{\mathbf{Q}}_k^H \mathbf{P}_k^o$

$\quad \mathbf{A}_i = \frac{\mathbf{P}_k^o - \overline{\mathbf{Q}}_k \underline{\mathbf{Q}}_k}{\| \mathbf{P}_k^o - \overline{\mathbf{Q}}_k \underline{\mathbf{Q}}_k \|_F}$

end for

$\underline{\mathbf{Q}} = \sqrt{N_s} \frac{\underline{\mathbf{Q}}}{\| \overline{\mathbf{Q}} \underline{\mathbf{Q}} H \|_F}$, when $\zeta = 1$

return $\overline{\mathbf{Q}}, \underline{\mathbf{Q}}$

Precoder: $\overline{N} = \overline{N}_t, \mathbf{P}^o = \mathbf{V}^o, \mathbf{\Phi} = \mathbf{\Gamma}_{\mathbf{y}_k^r}^{\frac{1}{2}}, \zeta = 1,$
$\overline{\mathbf{Q}} = \overline{\mathbf{V}}$, and $\underline{\mathbf{Q}} = \underline{\mathbf{V}}$

Receive filter: $\overline{N} = \overline{N}_r, \mathbf{P}^o = \mathbf{R}^{oH}, \mathbf{\Phi} = \mathbf{\Gamma}_{\mathbf{y}_k}^{\frac{1}{2}}, \zeta = 0, ,$
$\overline{\mathbf{Q}} = \overline{\mathbf{R}}$, and $\underline{\mathbf{Q}} = \underline{\mathbf{R}}$

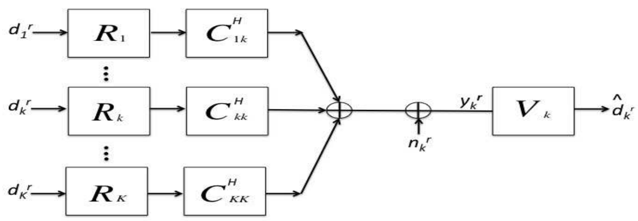

Fig. 2. Equivalent system for modeling reverse interference channel.

The set of candidate beamforming vectors are acquired from different dictionary sets. We compare the performance for both the proposed designs for different linear dictionaries as they consists of orthogonal candidate vectors and hence practically easy to implement. Performance has been evaluated for both non-robust and robust designs for eigen beamforming, discrete Fourier transform(DFT), discrete Cosine transform(DCT), discrete Hadamard transform(DHT) and antenna selection beamforming linear dictionaries and is illustrated in simulation results. Eigen beamforming is most complex of all above mentioned dictionaries and consists of eigenvectors of $\mathbf{\Gamma}_{\mathbf{y}_k}$. However, it offers the perfect decomposition of optimal full-complexity matrices into hybrid baseband and RF units which is also proved in lemma-2 in [13], and hence offers a very good performance. Whereas, DFT, DCT and DHT dictionaries can be designed by using the columns of Fourier transform, Cosine transform and Hadamard transform matrices respectively. However, in case of implementation complexity, antenna selection beamforming method is considered to be simplest of all, as it can be implemented by using a switching circuit, where the dictionary consisting of columns of I_{nRx} can be considered.

IV. Performance Evaluation

In this section, we demonstrate the performance of both the proposed non-robust and robust mmWave system designs. We evaluate and compare the performance in terms of total transmit power and sum-rate for both the designs. Sum-rate for multi-user MIMO system is evaluated as, $\rho = \sum_{k=1}^{K} \log_2 |\mathbf{I} + \text{SINR}_k|$ where $\text{SINR}_k = \frac{\mathbf{R}_k \mathbf{C}_k \mathbf{V}_k \mathbf{V}_k^H \mathbf{C}_k^H \mathbf{R}_k^H}{\mathbf{R}_k \beta_k \mathbf{R}_k^H}$ and $\beta_k = \sigma_n^2 \mathbf{I} + \sum_{l=1}^{k} \mathbf{C}_{kl} \mathbf{V}_l \mathbf{V}_l^H \mathbf{C}_{kl}^H - \mathbf{C}_{kk} \mathbf{V}_k \mathbf{V}_k^H \mathbf{C}_{kk}^H$. The designs are tested for multiple beamforming techniques. Convergence of both the algorithm is also observed.

A. Simulation Parameters

The simulation environment is assumed to have $K = 4$ number of users, where data is sequentially processed in parallel $N_s = 2$ transmission streams. We assume $nTx = nRx = 20$ antennas at both transmitter and receiver side with $\overline{N}_t = \overline{N}_r = 5$ RF chains, achieving the reduced complexity by factor of 4 as compared to fully complex MIMO system. The channel assumed is Saleh-Valenzuela channel model with uniform linear antenna arrays with inter-element spacing as $\lambda/2$. The number of clusters and number of rays are assumed to be $N_{cl} \in \{4, 6\}$ and $N_{ray} = 5$ respectively. The stochastic errors in CSI are modeled as Gaussian random vector with zero mean and varying $\sigma_E^2 \in \{0.5, 1, 1.5\}$. BPSK modulation scheme is assumed for generation of data. Multiple beamforming techniques are considered for performance evaluation as discussed in III-B.

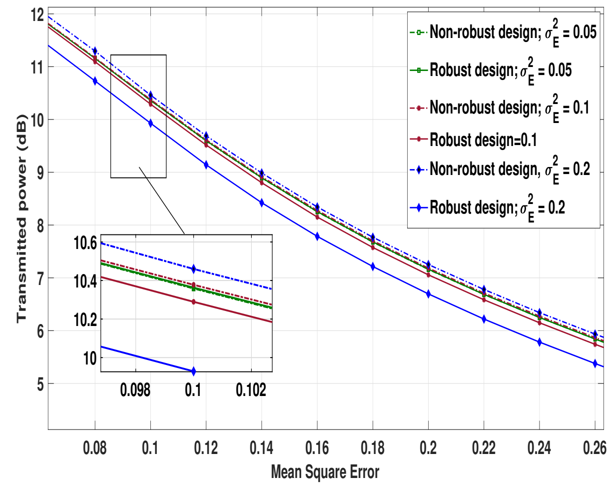

Fig. 3. Comparison of total transmit power performance for non-robust (dashed line) and robust (solid line) against varying MSE.

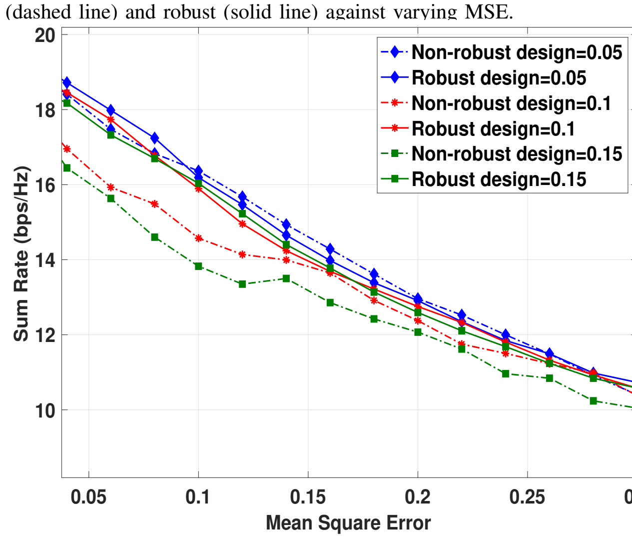

Fig. 4. Comparison of sum-rate performance Vs MSE for robust(solid line) and non-robust(dashed line).

B. Simulation Results

In Fig. 3 and 4, we compare the performance of proposed designs in terms of total transmit power and sum-rate respectively over a range of MSE for varying error variance σ_E^2. As seen in Fig. 3, power requirement decreases with increasing MSE. It is also observed that robust scheme require lesser transmit power as compared to non-robust scheme. However as σ_E^2 increases, the difference between power driven by both the schemes increases. From Fig. 4, sum-rate is higher for robust system as compared to non-robust design for all error variance, even though it reduces with increasing MSE. Hence from above results, it is observed that the robust scheme performs better than the other. The robust design is observed to perform better as its optimization problem already considers

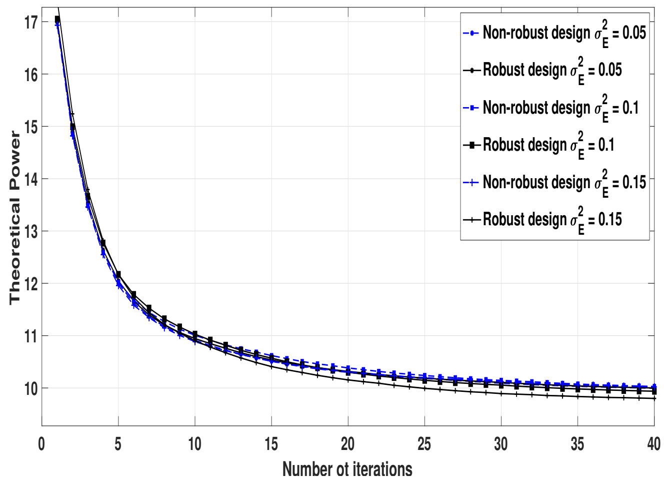

Fig. 5. Convergence behavior of non-robust Vs robust design for K=4.

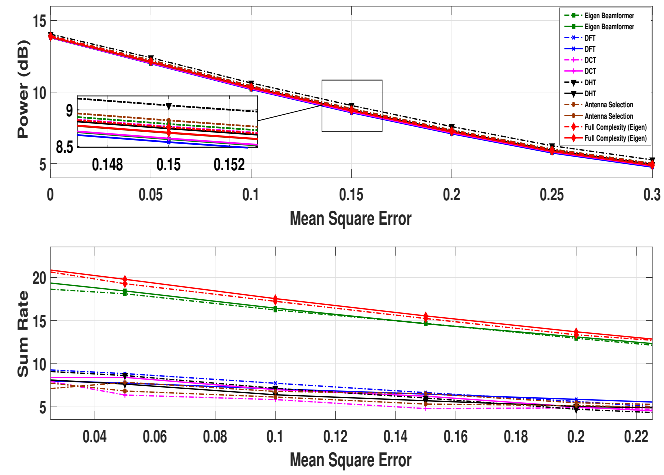

Fig. 6. Transmit power and sum-rate for robust(solid line) and non-robust design(dashed line) Vs MSE for different dictionaries.

the channel impairments hence obtaining better solutions and combating the effect of errors in available CSI knowledge as compared to non-robust scheme, where channel impairments are neglected during the design. We also observe the convergence of the proposed iterative algorithm and compare the difference in convergence curve for robust and non-robust systems that is plotted against varying error variance in Fig. 5. It is observed that both the algorithms converge in around 15 iterations with robust algorithm converging slightly faster than non-robust design. Simulations for various dictionaries for both non-robust and robust schemes has been performed for range of MSE with given error variance, $\sigma_E^2 = 0.1$ and comparison results are shown in the Fig. 6. Again, it is observed that, the eigen dictionary outperforms over other dictionaries in both the cases due to its perfect decomposition into hybrid matrices from optimal solutions. Other dictionaries show good performance over both the designs with slightly better results for robust design. But, considering their simple implementation they can be chosen for applications having non-critical criterion.

V. Conclusion

In this paper, we proposed two low-complexity analog-digital hybrid mmWave communication system designs, (non-robust and robust), where, complexity for both the designs was reduced by a factor of 4 as compared to full-complexity traditional MIMO system. We proposed two hybrid transceiver designs for K-user interference channel assuming the availability of different CSI knowledge at mmWave frequencies. The robust design demonstrates the resilience to erroneous CSI as compared to non-robust design. The transceivers were designed by formulating and solving total transmit power minimizing optimization problem constrained on sum-MSE for both the schemes. Convergence of the proposed algorithm to a limit was demonstrated. We adopted OMP-based sparse approximation technique to obtain the RF-baseband decomposition of the optimal transmit and receiver processing matrices. Performance for both the schemes was compared for different system parameter values. We also tested the proposed design for different RF processing methods for both the proposed transceiver designs. Simulation results show that the proposed robust hybrid system outperforms the non-robust hybrid design in the presence of errors in the available CSI.

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