Low-Complexity Transceiver Design for Multi-User Millimeter Wave Communication Systems under Imperfect CSI

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***Abstract*—In this paper, we present a robust analog-digital hybrid transceiver design for millimeter wave (mmWave) communication systems operating in a K-user MIMO interference channel. The precoder design is based on minimizing sum-mean square error (sum-MSE) under the assumption that the transceivers possess only imperfect information about the channel state. The channel state information (CSI) error is modelled as gaussian random vectors. In order to reduce the hardware complexity of the transceivers resulting from the use of large number of antennas in mmWave systems, we propose reduced-complexity designs using sparse approximation techniques. We show that the proposed algorithm convergence to a limit even though the global convergence is hard to prove due to the non-convex nature of the overall design problem. Furthermore, we evaluate the performance of proposed algorithm with various dictionaries. We numerically compare the performance of the proposed robust design with existing non-robust designs and the results demonstrate the resilience of the proposed algorithm to errors in the CSI.**

I. Introduction

Millimeter wave (mmWave) communication is a promising technology that can provide multigigabit-per-second data rate for future cellular systems. The main reason for the tremendous interest this technology has been gaining is the availability of huge unlicensed spectrum (3-300GHz). Harnessing this under-utilized spectrum opens up massive bandwidth for next generation mobile communication networks [1].

However, when compared to the current cellular systems, there are various challenges that accompany the potential benefits, such as free space path loss, less significant scattering due to ten fold increase in carrier frequency and pronounced coverage and blockage holes due to weaker non line-of-sight paths [2]. Thus, high-gain electronically steerable directional antennas are required to achieve high signal-noise-ratio (SNR), which can be obtained by beamforming or precoding data on large-scale antenna arrays, making MIMO a key technique mmWave systems. Since the size of antennas at mmWave frequency is small, large number of antennas can be packed in small volumes. On the other hand, dedicating a separate radio frequency (RF) chain for each antenna will lead to high cost and power consumption unlike in the case of traditional MIMO. One way to overcome this limitation is the use of hybrid precoding (analog-digital processing) algorithms [3], [4]. Hybrid precoding refers to decomposition of system into analog RF beamformer that aims to obtain large antenna gains and digital baseband MIMO processor that performs in low-dimensional equivalent channels. Recently in [3], hybrid precoding for point to point mmWave system was formulated as a spatially sparse precoding problem, and in [11] authors formulated the hybrid transceiver design as a compressed sensing mathematical problem for a similar system. In [10] design for limited feedback hybrid precoder was studied for mmWave system for downlink channel. Hybrid processing using orthogonal matching pursuit (OMP)-based sparse approximation technique was presented in [4].

But, the works cited above assume that the CSI is perfectly known by both the transmitters and receivers. However, the CSI available with the system is not always perfect due to various factors that can introduce errors such as estimation, feedback delays, quantization etc. Hence, the precoders and receive filters that are designed assuming the availability of perfect CSI are not likely to achieve desired performance in the presence of CSI errors. Effect of imperfect CSI on the performance of MIMO systems is discussed in [12]. This motivates us to design robust tranceiver algorithms which are resilient to erroneous CSI in mmWave systems.

In this paper, we propose a reduced-complexity hybrid precoder and receive filter design that provides robust performance in the presence of CSI errors. More specifically, we consider a multi-user interference channel with stochastic CSI errors. In the literature, such robust designs have been studied for conventional MIMO systems. The work in [13] considered a robust iterative algorithm for joint tranceiver design using min-max with probabilistically modeled CSI. Here, precoder and receive filter are jointly designed by minimizing sum-MSE of all the users. Resultant optimal precoders and receive filters have high hardware and computational complexity due to large number of antennas to be used in mmWave systems. However, we partition the overall processing to digital baseband and anlog RF processing by using sparse approximation. This partition makes reduced-complexity implementation possible. The corresponding optimization problem is not a convex problem and hence the global solution is not guaranteed. However, we show that proposed solution converges to an optimal point. We study the performance of the proposed design for different dictionaries composed of Eigen Beamforming, Discrete Hadamard Transform (DHT), Antenna Selection, Discrete Cosine Transform (DCT) and Discrete Fourier Transform (DFT)).

The remainder of this paper is organized as follows. Sec. II describes the system model for the proposed robust multi-user hybrid baseband/RF processing for mmWave interference channel with imperfect CSI. The design for hybrid OMP based Tx and Rx has been discussed in Sec. III and Sec. IV presents the simulation results and comparison to non-robust design. Finally, conclusions are given in Sec. V.

Notations: Throughout this paper, we use bold-faced lowercase letters to denote column vectors and bold-faced uppercase letters to denote matrices. $\underline{X}$ implies that the variable X corresponds to the baseband block and $\overline{X}$ implies that the variable X corresponds to the RF block. $\mathbf{C}_{jk}$ is the $(j,k)^{th}$ entry of $\mathbf{C}$. $\mathrm{tr}(\cdot)$ denotes the trace operator and $\mathbb{E}\{\cdot\}$ is the expectation operator.

II. System Model

We consider a $K-$user interference channel as shown in Fig.1, where transmitters and receivers are equipped with analog-digital hybrid precoders and combiners, respectively. A hybrid unit processes the data in two sequential phases, *viz.*, analog RF beamforming and digital baseband processing [3]. Each transmitter transmits N_S symbols over nTx antennas and each receiver is equipped with nRx antennas. The signal transmitted by the kth transmitter is denoted by the N_S-dimensional column vector $\mathbf{d}_k$. The number of RF chains associated with each transmitter and receiver are $\overline{N}_t$ and $\overline{N}_r$, respectively, and $N_s \leq \overline{N}_t < nTx$ and $N_s \leq \overline{N}_r < nRx$. The $\overline{N}_t \times N_s$ matrix $\underline{\mathbf{V}}_k$ denotes the digital baseband precoder and the $N_t \times \overline{N}_t$ matrix $\overline{\mathbf{V}}_k$ denotes the RF beamformer at the kth transmitter. On the receiver side, the signal vector received by the kth receiver is passed through a RF beamfomer denoted by $\overline{N}_r \times nRx$ matrix $\overline{\mathbf{R}}_k$ followed by a baseband combiner denoted by the $N_s \times \overline{N}_r$ matrix $\underline{\mathbf{R}}_k$. The output at the k^{th} RF beamformer is $\mathbf{z}_k$, where, $\mathbf{z}_k = \overline{\mathbf{R}}_k^H \mathbf{y}_k$. Let $\mathbf{V}_k^o$ and $\mathbf{R}_k^o$ denote the optimal precoder and receive filter in the conventional MIMO interference channel. Then, we design the hybrid filters to satisfy the following: $\mathbf{V}_k^o = \underline{\mathbf{V}}_k\overline{\mathbf{V}}_k$ and $\mathbf{R}_k^o = \underline{\mathbf{R}}_k^H\overline{\mathbf{R}}_k^H$.

We assume the transmitters possess only imperfect knowledge of the channel state. Specifically, the actual CSI can be modelled as

$$\mathbf{C} = \widehat{\mathbf{C}} + \mathbf{\Delta}, \tag{1}$$

where $\widehat{\mathbf{C}}$ is the estimated CSI available with the transmitters, and $\mathbf{\Delta} \sim \mathcal{N}(0, \sigma_E^2)$ denote the corresponding error in the CSI. The additive noise at the receivers is white gaussian noise, i.e., $\mathbf{n}_k \in \mathbb{C}^{nRx \text{ x } 1}$ with $\mathbf{n}_k \sim \mathcal{CN}(0, \sigma^2\mathbb{I}_{nRx})$ for $k = 1, 2, \cdots, K$. Following the above system model, the estimate of the transmitted data can be given by,

$$\widehat{\mathbf{d}}_k = \underline{\mathbf{R}}_k^H \mathbf{z}_k$$

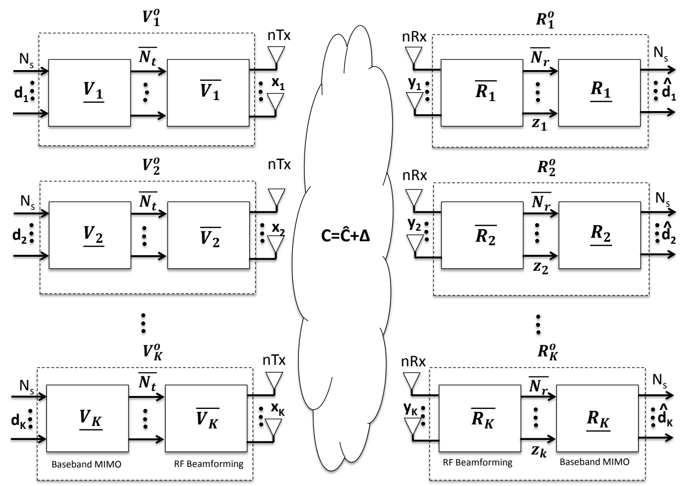

Fig. 1. Robust hybrid MIMO processor system design with K user interference channel

$$\begin{aligned}\widehat{\mathbf{d}}_k &= \underline{\mathbf{R}}_k^H\overline{\mathbf{R}}_k^H(\widehat{\mathbf{C}}_{kk} + \mathbf{\Delta})\overline{\mathbf{V}}_k\underline{\mathbf{V}}_k\mathbf{s}_k \\ &+ \underline{\mathbf{R}}_k^H\overline{\mathbf{R}}_k^H\sum_{j\neq k}^{K}(\widehat{\mathbf{C}}_{kj} + \mathbf{\Delta})\overline{\mathbf{V}}_j\underline{\mathbf{V}}_j\mathbf{s}_j \\ &+ \underline{\mathbf{R}}_k^H\overline{\mathbf{R}}_k^H\mathbf{n}_k,\end{aligned} \tag{2}$$

where the matrix $\mathbf{C}_{ij}$ denotes the channel gain between ith transmitter and jth receiver.

III. Robust hybrid design with stochastic CSI error

In this section, we present the design of hybrid precoding matrices $\overline{\mathbf{V}}_k$ and $\underline{\mathbf{V}}_k$ and hybrid receive filter matrices $\overline{\mathbf{R}}_k$ and $\underline{\mathbf{R}}_k$ for multiuser MIMO interference channel with stochastic CSI error. We first design the robust optimal precoder and receive filter $\mathbf{V}_k^o$ and $\mathbf{R}_k^o$ respectively, by minimising sum-MSE. Subsequently, we obtain the robust hybrid precoders and receive filters from the optimal matrices using the OMP-based sparse approximation technique.

A. Robust Optimal Precoder and Receive Filter Design

We design the optimal transmit precoding and receiving filter matrices $\mathbf{V_k^o}$ and $\mathbf{R_k^o}$ in order to ensure that their performance is robust in the presence of stochastic errors in the available CSI. This is acheived by considering expected values of the optimization objective and constraints. The expectation is with respect to the CSI error variables, thus proving immunity to the performance degradation resulting form the CSI errors. In the presence of CSI error that can be modeled as above, the MSE at the k^{th} user is given by,

TABLE I

Iterative algorithm computing $\mathbf{V}^o$ and $\mathbf{R}^o$ for robust MSE optimization
1. Initialize n = 0, $\mathbf{V}_k(0) \quad \forall k \in \{1,..K\}$
2. Update $\mathbf{R}_k$ next iteration using,
$\mathbf{R}_k(n+1) = \mathbf{V}_k^H(n)\widehat{\mathbf{C}}_{kk}^H\Big(\sum_{i=1}^K \widehat{\mathbf{C}}_{ki}\mathbf{V}_{i(n)}\mathbf{V}_{i(n)}^H\widehat{\mathbf{C}}_{ki}^H + \sigma_n^2\mathbf{I} + \sigma_E^2\sum_{i=1}^K\big(\mathbf{V}_{i(n)}\mathbf{V}_i^H(n)\big)\Big)^{-1}$
3. Solving
$\mathbf{V}_k(\tilde{\lambda_k}) = \Big[\sum_{i=1}^K \widehat{\mathbf{C}}_{ik}^H\mathbf{R}_i^H(n+1)\mathbf{R}_i(n+1)\widehat{\mathbf{C}}_{ik} + \tilde{\lambda_k}\mathbf{I} + \sigma_E^2\sum_{k=1}^K \mathrm{tr}\big(\mathbf{R}_k^H(n+1)\mathbf{R}_k(n+1)\big)\mathbf{I}\Big]^{-1}\widehat{\mathbf{C}}_{kk}^H\mathbf{R}_k(n+1)^H$
$\lambda_k(n+1) = \Big[\{\tilde{\lambda_k} \mid \text{such that } \mathrm{tr}\big(\mathbf{V}_k(\tilde{\lambda_k})^H\big)\mathbf{V}_k(\tilde{\lambda_k}) = P_k\}\Big]_+$
$\mathrm{tr}\Big[\mathbf{R}_k(n+1)\widehat{\mathbf{C}}_{kk}\Big[\sum_{i=1}^K \widehat{\mathbf{C}}_{ik}^H\mathbf{R}_i(n+1)^H\mathbf{R}_i(n+1)\widehat{\mathbf{C}}_{ik} + \tilde{\lambda_k}\mathbf{I} + \sigma_E^2\sum_{k=1}^K \mathrm{tr}\big(\mathbf{R}_k(n+1)^H\mathbf{R}_k(n+1)\big)\mathbf{I}\Big]^{-1}\Big]$
$\Big[\Big[\sum_{i=1}^K \widehat{\mathbf{C}}_{ik}^H\mathbf{R}_i(n+1)^H\mathbf{R}_i(n+1)\widehat{\mathbf{C}}_{ik} + \tilde{\lambda_k}\mathbf{I} + \sigma_E^2\sum_{k=1}^K \mathrm{tr}\big(\mathbf{R}_k(n+1)^H\mathbf{R}_k(n+1)\big)\mathbf{I}\Big]^{-1}\widehat{\mathbf{C}}_{kk}^H\mathbf{R}_k^H(n+1)\Big] - P_k = 0$
4. Update,
$\mathbf{V}_k(n+1) = \Big[\sum_{i=1}^K \widehat{\mathbf{C}}_{ik}^H\mathbf{R}_i(n+1)^H\mathbf{R}_i(n+1)\widehat{\mathbf{C}}_{ik} + \lambda_k(n+1)\mathbf{I} + \sigma_E^2\sum_{k=1}^K \mathrm{tr}\big(\mathbf{R}_k^H(n+1)\mathbf{R}_k(n+1)\big)\mathbf{I}\Big]^{-1}\widehat{\mathbf{C}}_{kk}^H\mathbf{R}_k^H(n+1)$
5. Repeat 2,3,4 until optimization is achieved.

$$
\begin{aligned}
\mathrm{MSE}_k &= \mathbb{E}\{\|\widehat{\mathbf{d}}_k - \mathbf{d}_k\|^2\} \\
&= \mathbb{E}\Big[\mathrm{tr}\Big(\Big(\big(\mathbf{R}_K(\widehat{\mathbf{C}}_{kk}+\mathbf{\Delta})\mathbf{V}_k - \mathbf{I}\big)\mathbf{d}_k \\
&+ \mathbf{R}_k\sum_{i=1}^K(\widehat{\mathbf{C}}_{ki}+\mathbf{\Delta})\mathbf{V}_i\mathbf{d}_i + \mathbf{R}_k\mathbf{n}_k\Big) \\
&\Big(\big(\mathbf{R}_K(\widehat{\mathbf{C}}_{kk}+\mathbf{\Delta})\mathbf{V}_k - \mathbf{I}\big)\mathbf{d}_k \\
&+ \mathbf{R}_k\sum_{i=1}^K(\widehat{\mathbf{C}}_{ki}+\mathbf{\Delta})\mathbf{V}_i\mathbf{d}_i + \mathbf{R}_k\mathbf{n}_k\Big)^H\Big)\Big] \\
&= \mathrm{tr}\Big(\mathbf{R}_k\sum_{i=1}^K(\widehat{\mathbf{C}}_{ki}\mathbf{V}_i\mathbf{V}_i^H\widehat{\mathbf{C}}_{ki}^H)\mathbf{R}_k^H \\
&- (\mathbf{R}_k\widehat{\mathbf{C}}_{kk}\mathbf{V}_k + \mathbf{V}_k^H\widehat{\mathbf{C}}_{kk}^H\mathbf{R}_k^H) + \sigma_n^2\mathbf{R}_k\mathbf{R}_k^H + \mathbf{I}\Big) \\
&+ \mathbb{E}\Big[\mathrm{tr}\Big(\mathbf{R}_\mathrm{k}(\sum_{\mathrm{i=1}}^{\mathrm{K}}\mathbf{\Delta}\mathbf{V}_\mathrm{i}\mathbf{V}_\mathrm{i}^\mathrm{H}\mathbf{\Delta}^\mathrm{H})\mathbf{R}_\mathrm{k}^\mathrm{H}\Big)\Big].
\end{aligned} \tag{3}
$$

We need to further simplify the expression for MSE in order to proceed with the computation of optimal matrices. A Lemma in [6] states that, for any random matrix $\mathbf{X}$ with $\mathbb{E}\{\mathbf{X}\mathbf{X}^H\} = \sigma^2\mathbf{I}$, and matrices $\mathbf{U}$ and $\mathbf{V}$ of appropriate dimensions, the following equality holds:

$$
\begin{aligned}
\mathbb{E}[tr(\mathbf{X}\mathbf{U}\mathbf{X}^H\mathbf{V})] &= \mathbb{E}[tr(\mathbf{X}^H\mathbf{V}\mathbf{X}\mathbf{U})] \\
&= \sigma^2 tr(\mathbf{U})tr(\mathbf{V}).
\end{aligned} \tag{4}
$$

Employing this result, we can simplify the last term in (3) as

$$
\begin{aligned}
&\mathbb{E}\Big\{\mathrm{tr}\Big(\sum_{i=1}^K \mathbf{R}_k\mathbf{\Delta}\mathbf{V}_i\mathbf{V}_i^H\mathbf{\Delta}^H\mathbf{R}_k^H\Big)\Big\} \\
&\quad = \sum_{i=1}^K \sigma_E^2\mathrm{tr}(\mathbf{V}_i\mathbf{V}_i^H)\mathrm{tr}(\mathbf{R}_k^H\mathbf{R}_K).
\end{aligned} \tag{5}
$$

Based on the preceding developments, the sum-MSE for $K-$user interference channel in the presence of imperfect CSI can be expressed as

$$
\begin{aligned}
\sum_{k=1}^K \mathrm{MSE_k} = \mathrm{tr}\Big(&\sum_{k=1}^K \mathbf{R}_k\sum_{i=1}^K \widehat{\mathbf{C}}_{ki}\mathbf{V}_i\mathbf{V}_i^H\widehat{\mathbf{C}}_{ki}^H\mathbf{R}_k^H \\
&- \sum_{k=1}^K(\mathbf{R}_k\widehat{\mathbf{C}}_{kk}\mathbf{V}_k + \mathbf{V}_k^H\widehat{\mathbf{C}}_{kk}^H\mathbf{R}_k^H) \\
&+ \sum_{k=1}^K \sigma_n^2\mathbf{R}_k\mathbf{R}_k^H + K\mathbf{I}\Big) \\
&+ \sum_{k=1}^K\sum_{i=1}^K \sigma_E^2\mathrm{tr}(\mathbf{V}_i\mathbf{V}_i^H)\mathrm{tr}(\mathbf{R}_k^H\mathbf{R}_k).
\end{aligned} \tag{6}
$$

We design the optimal transceiver by minimizing the sum-MSE under the constraint on total transmit power. Thus, the optimization problem can be mathematically expressed as,

$$
\begin{aligned}
&\min_{\{\mathbf{V_k}\},\{\mathbf{R_k}\}} \quad \sum_{k=1}^K \mathrm{MSE}_k \\
&\text{subject to:} \quad \mathrm{tr}(\mathbf{V_k}^H\mathbf{V_k}) \le P_k, \forall k \in \{1,...K\},
\end{aligned} \tag{7}
$$

where P_k is the upper limit on the transmit power of the kth transmitter. This optimization problem can be solved using the Karush-Kuhn-Tucker conditions to be satisfied by the optimal solution [14]. Towards this end, the Lagrangian associated with the optimization problem can be expressed as

$$
\begin{aligned}
L(V_k, R_k, \lambda_k) &= \sum_{k=1}^{K} \mathrm{MSE}_k + \sum_{k=1}^{K} \lambda_k[\mathrm{tr}(\mathbf{V}_\mathrm{k}^\mathrm{H}\mathbf{V}_\mathrm{k}) - \mathrm{P_k}] \\
&= \mathrm{tr}\Big(\sum_{\mathrm{k}=1}^{\mathrm{K}} \mathbf{R}_\mathrm{k}(\sum_{\mathrm{i}=1}^{\mathrm{K}} \widehat{\mathbf{C}}_\mathrm{ki}\mathbf{V}_\mathrm{i}\mathbf{V}_\mathrm{i}^\mathrm{H}\widehat{\mathbf{C}}_\mathrm{ki}^\mathrm{H})\mathbf{R}_\mathrm{k}^\mathrm{H} \\
&\quad - \sum_{k=1}^{K}(\mathbf{R}_k\widehat{\mathbf{C}}_{kk}\mathbf{V}_k + \mathbf{V}_k^H\widehat{\mathbf{C}}_{kk}^H\mathbf{R}_k^H) \\
&\quad + \sum_{k=1}^{K}\sigma_n^2\mathbf{R}_k\mathbf{R}_k^H + K\mathbf{I}\Big) \\
&\quad + \sum_{k=1}^{K}\sum_{i=1}^{K}\sigma_E^2\mathrm{tr}(\mathbf{V}_\mathrm{i}\mathbf{V}_\mathrm{i}^\mathrm{H})\mathrm{tr}(\mathbf{R}_\mathrm{k}^\mathrm{H}\mathbf{R}_\mathrm{k}) \\
&\quad + \sum_{k=1}^{K}\lambda_k[\mathrm{tr}(\mathbf{V}_\mathrm{k}^\mathrm{H}\mathbf{V}_\mathrm{k}) - \mathrm{P_k}],
\end{aligned} \tag{8}
$$

where λ_k, $k = 1, 2, \cdots, K$ are the Lagrangian variables. It can be observed that sum-MSE function is not jointly convex in the otpimization varibales, but it is convex in $\{\mathbf{V}_k\}$ for fixed values of $\{\mathbf{R}_k\}$ and vice versa. Based on this observation, we obtain the soltuin to the optimization problem by the coordinate descent method, wherein the minimization is performed with respect to one variable while keeping other variables fixed. Thus the optimal values for $\mathbf{V^o}$ and $\mathbf{R^o}$ are obtained iteratively. Considering minimization with respect to $\mathbf{V_k}$ while keeping $\{\mathbf{R}_k\}$ fixed, we set

$$
\frac{\partial L}{\partial \mathbf{V}_k^*} = 0. \tag{9}
$$

From the condition given above, we have

$$
\begin{aligned}
\mathbf{V}_k = \Big[\sum_{i=1}^{K}\widehat{\mathbf{C}}_{ik}^H\mathbf{R}_i^H(n+1)\mathbf{R}_i(n+1)\widehat{\mathbf{C}}_{ik} + \lambda_k\mathbf{I} + \sigma_E^2 \\
\sum_{k=1}^{K}\mathrm{tr}\big(\mathbf{R}_\mathrm{k}^\mathrm{H}(\mathrm{n}+1)\mathbf{R}_\mathrm{k}(\mathrm{n}+1)\big)\mathbf{I}\Big]^{-1}\widehat{\mathbf{C}}_\mathrm{kk}^\mathrm{H}\mathbf{R}_\mathrm{k}^\mathrm{H}(\mathrm{n}+1).
\end{aligned} \tag{10}
$$

Similarly, we minimize with respect to $\mathbf{R_k}$ while keeping other variables fixed by setting

$$
\frac{\partial L}{\partial \mathbf{R}_\mathbf{k}^*} = 0. \tag{11}
$$

From the above-given condition, we have

$$
\begin{aligned}
\mathbf{R}_k = \mathbf{V}_k^H\widehat{\mathbf{C}}_{kk}^H\big(\sum_{i=1}^{K}\widehat{\mathbf{C}}_{ki}\mathbf{V}_i\mathbf{V}_i^H\widehat{\mathbf{C}}_{ki}^H \\
+ \sigma_n^2\mathbf{I} + \sigma_E^2\sum_{i=1}^{K}(\mathbf{V}_i\mathbf{V}_i^H)\big)^{-1}.
\end{aligned} \tag{12}
$$

The complete iterative algorithm for robust design based on the preceding developments is provided in Table-I. Since the objective is non-increasing with each iteration, and it is lower-bounded, the objective tends to limit as the number of iteration increases. However, the global optimality is not guaranteed since the original problem is not convex.

TABLE II

OMP-based iterative algorithm for robust MSE optimization
Require $\mathbf{P}_k^o, \mathbf{\Phi}, \mathbf{S}_{BF}$
1: $\overline{\mathbf{Q}}_k = [\ ]$
2: $\mathbf{A}_0 = \mathbf{P}_k^o$
3: for $i = 1$ to $\overline{N}$ do
4: $\mathbf{\Psi}_{i-1} = (\mathbf{\Phi}\mathbf{S}_{BF})^H(\mathbf{\Phi}\mathbf{A}_{i-1})$
5: $l = arg\ max_{m=1\ldots M}(\mathbf{\Psi}_{i-1}\mathbf{\Psi}_{i-1}^H)_{m,m}$
6: $\underline{\mathbf{Q}}_k = [\overline{\mathbf{Q}}_k \| \mathbf{\Phi}(:,k)]$
7: $\underline{\mathbf{Q}}_k = (\overline{\mathbf{Q}}_k^H\overline{\mathbf{Q}}_k)^{-1}\overline{\mathbf{Q}}_k^H\mathbf{P}_k^o$
8: $\mathbf{A}_i = \frac{\mathbf{P}_k^o - \overline{\mathbf{Q}}_k\underline{\mathbf{Q}}_k}{\\|\mathbf{P}_k^o - \overline{\mathbf{Q}}_k\underline{\mathbf{Q}}_k\\|_F}$
9: end for
10: $\underline{\mathbf{Q}} = \sqrt{N_s}\frac{\mathbf{Q}}{\\|\overline{\mathbf{Q}}\underline{\mathbf{Q}}H\\|_F}$
11: return $\overline{\mathbf{Q}}, \underline{\mathbf{Q}}$
Precoder: $\overline{N} = \overline{N}_t, \mathbf{P}^o = \mathbf{V}^o, \mathbf{\Phi} = \mathbf{\Gamma}_{\mathbf{y}_k^r}^{\frac{1}{2}}$,
$\overline{\mathbf{Q}} = \overline{\mathbf{V}}$, and $\underline{\mathbf{Q}} = \underline{\mathbf{V}}$
Receive filter: $\overline{N} = \overline{N}_r, \mathbf{P}^o = \mathbf{R}^{oH}, \mathbf{\Phi} = \mathbf{\Gamma}_{\mathbf{y}_k}^{\frac{1}{2}}$,
$\overline{\mathbf{Q}} = \overline{\mathbf{R}}$, and $\underline{\mathbf{Q}} = \underline{\mathbf{R}}$

B. Hybrid OMP-Based Robust Transceiver Design

We use the OMP sparse approximation technique in order to decompose the optimal matrices $\{\mathbf{V}_k^o\}$ and $\{\mathbf{R}_k^o\}$ obtained in previous subsection into their corresponding baseband and RF processing matrices. OMP algorithm has been extensively studied in literature and has been used for multiple signal processing applications [7] - [9]. OMP-based sparse approximation problem has been formulated for multi-user mmWave communication system in [4]. We follow a similar approach to obtain the basedband and RF matrices for the robust design. The required hybrid processor can be realized by decomposing optimal sum-MSE solution as

$$
\mathbf{V}_k^o = \underline{\mathbf{V}}_k\overline{\mathbf{V}}_k \text{ and } \mathbf{R}_k^o = \underline{\mathbf{R}}_k^H\overline{\mathbf{R}}_k^H.
$$

For the receiver side, the optimal MMSE solution (12) can be rewritten as

$$
\mathbf{R}_k^o = \mathbf{\Gamma}_{\mathbf{y}_k}\mathbf{\Gamma}_{\mathbf{y}_k\mathbf{s}_k}^{-1}, \tag{13}
$$

where $\mathbf{\Gamma}_{\mathbf{y}_k} = \mathbb{E}[\mathbf{y}_k\mathbf{y}_k^H]$ and $\mathbf{\Gamma}_{\mathbf{y}_k\mathbf{d}_k} = \mathbb{E}[\mathbf{y}_k\mathbf{d}_k^H]$,
and the baseband receive filter matrix can be written as,

$$
\underline{\mathbf{R}}_k^o = \mathbf{\Gamma}_{\mathbf{z}_k}^{-1}\mathbf{\Gamma}_{\mathbf{z}_k\mathbf{d}_k} = (\overline{\mathbf{R}}_k^H\mathbf{\Gamma}_{\mathbf{y}_k}\overline{\mathbf{R}}_k)^{-1}\overline{\mathbf{R}}_k^H\mathbf{\Gamma}'_{\mathbf{y}_k\mathbf{d}_k} \tag{14}
$$

where $\mathbf{\Gamma}_{\mathbf{z}_k} = \mathbb{E}[\mathbf{z}_k\mathbf{y}_k^H]$ and $\mathbf{\Gamma}_{\mathbf{z}_k\mathbf{d}_k} = \mathbb{E}[\mathbf{z}_k\mathbf{d}_k^H]$. OMP sparse problem can be formulated as,

$$
\overline{\mathbf{R}}_k^o = \underset{\overline{\mathbf{R}}_k}{\mathrm{argmin}}\,\mathbb{E}\|\mathbf{d}_k - \underline{\mathbf{R}}_k^{o\,H}\overline{\mathbf{R}}_k\|^2, \tag{15}
$$

which can be rewritten as,

$$
\overline{\mathbf{R}}_k^o = \underset{\overline{\mathbf{R}}_k^o}{\mathrm{argmin}}\,\|\mathbf{\Gamma}_{\mathbf{y}_k}^{\frac{1}{2}}\mathbf{R}_k^o - \mathbf{\Gamma}_{\mathbf{y}_k}^{\frac{1}{2}}\overline{\mathbf{R}}_k\underline{\mathbf{R}}_k^o\|_F^2. \tag{16}
$$

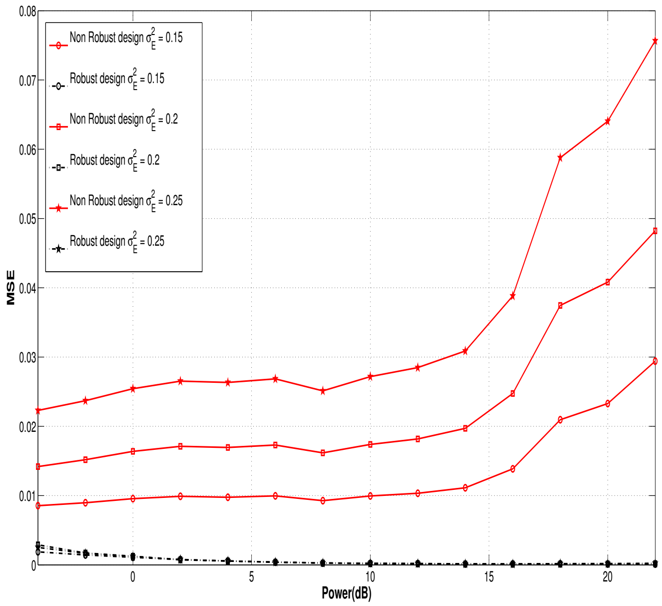

Fig. 2. MSE performance against varying transmit power for robust(dashed line) and non-robust design(solid line) when $K = 3, nTx = nRx = 20, \overline{N_t} = \overline{N_r} = 10$

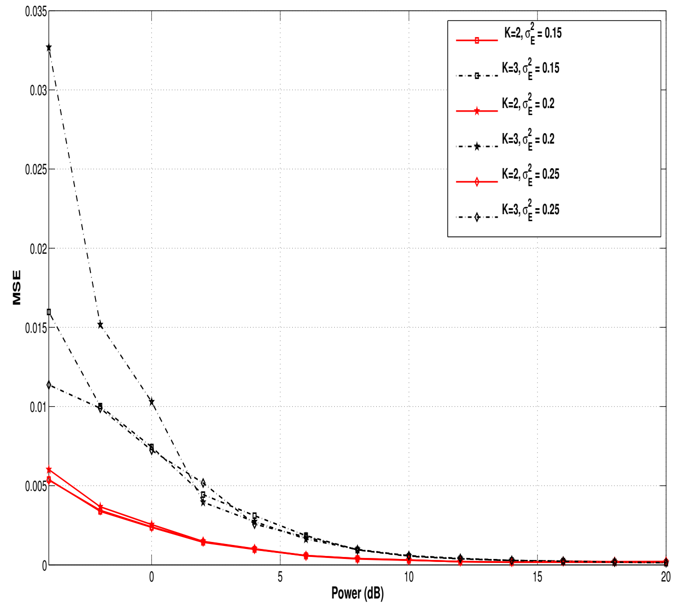

Fig. 3. MSE for K=2 (solid line) and K=3 (dashed line) against varying transmit power for robust design

With introduced dictionary $\mathbf{S}_{BF}$ the optimization problem can be rephrased as,

$$\begin{aligned}\underline{\tilde{\mathbf{R}}}_k^o &= \underset{\underline{\tilde{\mathbf{R}}}_k}{\operatorname{argmin}} \|\mathbf{\Gamma}_{\mathbf{y}_k}^{\frac{1}{2}} \mathbf{R}_k^o - \mathbf{\Gamma}_{\mathbf{y}_k}^{\frac{1}{2}} \mathbf{S}_{BF} \underline{\tilde{\mathbf{R}}}_k^o\|_F^2, \\ &\text{s.t. } \|\text{diag } (\underline{\tilde{\mathbf{R}}}_k^o \underline{\tilde{\mathbf{R}}}_k^{oH})\| = \overline{N}_r.\end{aligned} \tag{17}$$

Using the similar approach, the optimization problem for designing sparse precoder matrix can be written as,

$$\overline{\mathbf{V}}_k^o = \underset{\overline{\mathbf{V}}_k}{\operatorname{argmin}} \|\mathbf{\Gamma}_{\mathbf{y}_k^r}^{\frac{1}{2}} \mathbf{V}_k^o - \mathbf{\Gamma}_{y_k^r}^{\frac{1}{2}} \overline{\mathbf{V}}_k \underline{\mathbf{V}}_k^o\|_F^2 \tag{18}$$

$$\begin{aligned}\mathbf{V}_k^o &= \underset{\tilde{\mathbf{V}}_k}{\operatorname{argmin}} \|\mathbf{\Gamma}_{\mathbf{y}_k^r}^{\frac{1}{2}} \mathbf{V}_k^o - \mathbf{\Gamma}_{\mathbf{y}_k^r}^{\frac{1}{2}} \mathbf{S}_{BF} \tilde{\mathbf{V}}_k^o\|_F^2, \\ &\text{s.t. } \|\text{diag } (\underline{\tilde{\mathbf{V}}}_k^o \underline{\tilde{\mathbf{V}}}_k^{oH})\| = \overline{N}_t, \\ &\text{and } \|\underline{\tilde{\mathbf{V}}}_k\|_F^2 = \|\mathbf{V}_k^o\|_F^2.\end{aligned} \tag{19}$$

To obtain the optimal solution for the above problem at both the transmitter and receiver end, an OMP based iterative algorithm has been used, which is discussed in Table-II. It is a generalized algorithm for obtaining $\underline{\mathbf{V}}_k, \overline{\mathbf{V}}_k$ and $\underline{\mathbf{R}}_k, \overline{\mathbf{R}}_k$ by passing the appropriate parameters. The input parameter $\mathbf{S}_{BF}$ represents the set of candidate beamforming vectors. These candidate vectors can be acquired from different linear or non-linear dictionary set. We compare the performance of our system for linear dictionaries as they consists of orthogonal candidate vectors and hence are practically easy to implement. Performance of the proposed system has been evaluated for Eigen Beamforming, Discrete Hadamard Transform (DHT), Antenna Selection, Discrete Cosine Transform (DCT), Discrete Fourier Transform (DFT) linear dictionaries.

IV. Simulation Results and Discussion

The performance of the proposed robust design for mmWave communication system have been examined through computer simulations. We compare the performance of proposed robust design to non-robust one in terms of sum-rate and MSE, where sum-rate for multi-user MIMO system was evaluated as, $sumrate = \sum_{k=1}^{K} \sum_{i=1}^{D_k} \log_2(1 + SINR_{k,i})$ where $SINR_{k,i} = \frac{\mathbf{R}_{ki}\mathbf{C}_{kk}\mathbf{V}_{ki}\mathbf{V}_{ki}^H\mathbf{C}_{kk}^H\mathbf{R}_{ki}^H}{\mathbf{R}_{ki}\mathbf{\Psi}_{ki}\mathbf{R}_{ki}^H}$ and $\mathbf{\Psi}_{ki} = \sigma_n^2\mathbf{I} + \sum_{l=1}^{k} \mathbf{C}_{kl}\mathbf{V}_l\mathbf{V}_l^H\mathbf{C}_{kl}^H - \mathbf{C}_{kk}\mathbf{V}_{ki}\mathbf{V}_{ki}^H\mathbf{C}_{kk}^H$. The design was tested for different beamforming techniques. We also observe the convergence for the proposed iterative algorithm. The channel model used in simulations is quasi static flat Rayleigh fading channel with stochastic errors in CSI knowledge. Simulations have been performed for N = 10000 data samples with $N_s = 2$ data streams.

In Fig. 2 and 3 the sum-rate and MSE performance w.r.t. various transmit powers for proposed robust design has been compared to that of non-robust design over various channel error variance in CSI knowledge. In Fig. 2, it has been observed that sum-rate for robust design increases rapidly with increasing transmit power, whereas, it is decreasing for non robust design over all error variance. Moreover we observe improved system performance for robust design with increasing error variance. Fig 3 also verifies the same behavior i.e MSE is decreasing with increasing transmit power in robust case whereas, in non robust MSE is observed to drastically increase with power. Convergence of the proposed iterative algorithm has been proved and is illustrated in Fig. 4 It is observed that the algorithm converges in less than five iterations. We simulated the system over various dictionaries for RF bramforming (Eigen Beamforming, Discrete Hadamard Transform

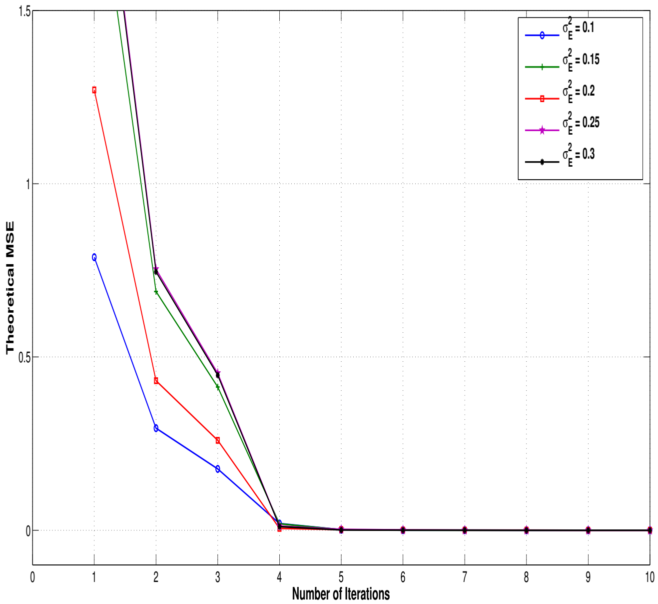

Fig. 4. Convergence behavior of robust design for transmit power = 20dB

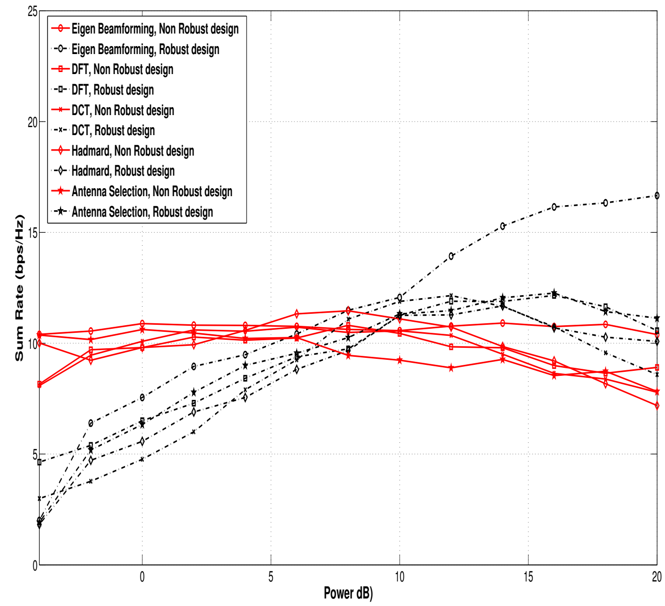

Fig. 5. Sum Rate performance against varying power for Robust(dashed line) and Non Robust design(solid line) when $K = 2, nTx = nRx = 16, \overline{N_t} = \overline{N_r} = 8$ for various dictionaries

(DHT), Antenna Selection, Discrete Cosine Transform (DCT), Discrete Fourier Transform (DFT) linear dictionaries) and results has been shown in the Fig. 5 Eigen value beamforming shows the best result over all other dictionaries. Whereas, other dictionaries show decent performance as compared to non-robust design, but considering their simple implementation they can be chosen for applications not demanding severely critical criterion.

V. Conclusion

In this paper, we proposed a robust low-complexity analog-digital hybrid MIMO processor that is resilient to erroneous CSI for a $K-$user interference channel operating at mmWave frequencies . A sum-MSE minimizing optimization problem was formulated under a constraint on total transmit power for the robust precoder and receive filter design. We showed that proposed algorithm converges to a limit. We adopted the analog-digital hybrid system in order to reduce the complexity by reducing the number of RF chains required in the system. OMP-based sparse approximation technique was used to obtain the RF-baseband decomposition of the optimal transmit and receiver processing matrices. We also tested the proposed design for different RF processing methods and observed that the eigen beamformer performs best among all the tested methods. The performance of the proposed robust design was compared to non-robust design for different values of system parameters. Simulation results shows that the proposed robust hybrid system outperforms the non-robust hybrid design in the presence of errors in the available CSI.

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